Revenue Management Without Commitment: Dynamic Pricing and Periodic Fire Sales*

Francesc Dilme and Fei Li†
University of Pennsylvania

November 12, 2012

Abstract

We consider a market with a profit-maximizing monopolist seller who has K identical goods to sell before a deadline. At each date, the seller posts a price and the quantity available but cannot commit to future offers. Over time, potential buyers with different reservation values enter the market. Buyers strategically time their purchases, trading off (1) the current price without competition and (2) a possibly lower price in the future with the risk of being rationed. We analyze equilibrium price paths and buyers' purchase behavior in which prices decline smoothly over the time period between sales and jump up immediately after a transaction. In equilibrium, high-value buyers purchase on arrival. Crucially, before the deadline, the seller may periodically liquidate part of his stock via a fire sale to secure a higher price in the future. Intuitively, these sales allow the seller to 'commit' to high prices going forward. The possibility of fire sales before the deadline implies that the allocation may be inefficient. The inefficiency arises from the scarce good being misallocated to low-value buyers, rather than the withholding inefficiency that is normally seen with a monopolist seller. **Keywords:** revenue management, commitment power, dynamic pricing, fire sales. **JEL Classification Codes:** D82, D83.

*We are grateful to George Mailath and Mallesh Pai for insightful instruction and encouragement. We also thank Aislinn Bohren, Simon Board, Eduardo Faingold, Hanming Fang, John Lazarev, Anqi Li, Steven Matthews, Guido Menzio, Andrew Postlewaite, Maher Said, Can Tian, Rakesh Vohra, Jidong Zhou and participants at UPenn Micro theory seminar for valuable comments. All remaining errors are ours. The latest version of this paper can be found at https://sites.google.com/site/lifei1019/home/job-market-paper.

†*Corresponding Author*: Fei Li, 160 McNeil Building, 3718 Locust Walk, Philadelphia, PA 19104-6297. Email: fei@econ.upenn.edu

When to buy your ticket is one of the most vexing decisions for travelers. Airlines bounce fares up and down regularly, sometimes several times in the same day. Sales come and go quickly, and availability of cheap seats on prime flights can be scarce. Travelers who wait for a better price can end up disappointed when prices keep rising. Travelers who jump on a fare at first search may end up angry if the price drops. It can be like playing poker against airlines.... For peak-season travel, fares start fairly high and then come down. Airlines start more-actively managing pricing on flights about three-to-four months before departure. That's also when shoppers start getting more active.

—– *Scott McCartney*[1]

1 Introduction

Many markets share the following characteristics: (1) goods for sale are (almost) identical, and all expire and must be consumed at a certain point of time, (2) the initial number of goods for sale is fixed in advance, and (3) consumers have heterogeneous reservation values and enter the market sequentially over time. Such markets include the airline, cruise-line, hotel and entertainment industries. The revenue management literature studies the pricing of goods in these markets, and these techniques are reported to be quite valuable in many industries, such as airlines (Davis (1994)), retailers (Friend and Walker (2001)), etc. The standard assumptions in this literature are that sellers have perfect commitment power and buyers are impatient. That is, buyers cannot time their purchases and sellers can commit to the future price path or mechanism. In contrast, this paper studies a revenue management problem in which buyers are *patient* and sellers are endowed with *no commitment power*.

We consider the profit-maximizing problem faced by a monopolist seller who has K identical goods to sell before a deadline. At any date, the seller posts a price and the quantity available (capacity control) but *cannot commit* to future offers. Over time, potential buyers with different reservation values (either high or low) privately enter the market. Each buyer has a single-unit demand and can time her purchase. Goods are consumed at the fixed deadline, and all trades happen before or at that point.

Our goal is to show that the seller can sometimes use fire sales before the deadline to credibly reduce his inventory and so charge higher prices

[1] WSJ blogs, June 28, 2012, "What's the Sweet Spot for Buying International Airline Tickets?"

in the future. We accordingly consider settings where the seller does not find it profitable to only sell at the deadline and then only to high-value buyers, with the accompanying possibility of unsold units. In such settings, we explore the properties of a pricing path in which, at the deadline, if the seller still has unsold goods, he sets the price sufficiently low that all remaining goods are sold for sure. For most of the time before the deadline, the seller posts the highest price consistent with high-value buyers purchasing immediately on arrival, and occasionally, he posts a fire sale price that is affordable to low-value buyers. By holding fire sales, the seller reduces his inventory quickly, and therefore, he can induce high-value buyers to accept a higher price in the future. Intuitively, these sales allow the seller to 'commit' to high prices going forward. Once the transaction happens, whether at the discount price or not, the seller's inventory is reduced, and the price jumps up instantaneously. Hence, in general, a highly fluctuating path of realized sales prices will appear, which is in line with the observations in many relevant industries.[2]

The suboptimality of only selling at the deadline to high-value buyers could occur for many reasons. For example, at the deadline, the seller may expect that there will be little effective high-value demand in the market. This may be because the arrival rate of high-value buyers is low, or because buyers may also leave the market without making a purchase, or because buyers face inattention frictions and so they may miss the deadline, which we discuss in detail below.

The equilibrium price path relies on the seller's lack of commitment and buyers' intertemporal concern. An intuitive explanation is as follows. At the deadline, due to the insufficient effective demand, the seller holding unsold goods sets a low price to clear his inventory, which is known as the last-minute deal.[3] Before the deadline, since a last-minute deal is expected to be posted shortly, buyers have the incentive to wait for the discount price.[4] However, waiting for a deal is risky due to competition at the low price, from both newly arrived high-value buyers and low-value ones who are only willing to pay a low price. By weighing the risk of losing the competition and

[2] For example, McAfee and te Velde (2008) find that airfares' fluctuation is too high to be explained by the standard monopoly pricing models.

[3] In the airline industry, sellers do post last-minute deals. See *Wall Street Journal*, March 15, 2002, "Airlines now offer 'last minute' fare bargains weeks before flights," by Kortney Stringer.

[4] In the airline industry, many travelers are learning to expect possible discounts in the future and strategically time their purchase. See the *Wall Street Journal*, July 2002, "A Holiday for Procrastinators: Booking a Last-Minute Ticket," by Eleena de Lisser.

so the deal, a high-value buyer is willing to make her purchase immediately at a price higher than the discount one. We name the highest price she is willing to pay to avoid the competition as her reservation price. For any such high-value buyer, her reservation price is decreasing in time, since the arrival of competition shrinks as the deadline approaches, and decreasing in the current inventory size, since the probability that she will be rationed at deal time depends on the amount of remaining goods. To maximize his profit, the seller posts the high-value buyer's reservation price for most of the time and, at certain times before the deadline, may hold fire sales to reduce his inventory and to charge a higher price in the future.

Figure 1 illustrates this idea in the simplest case with only two items for sale at the beginning. Suppose the seller serves high-value buyers only before the deadline, allowing discounts at the deadline only. Conditional on the inventory size, the price declines in time. The high-value buyer's acceptable price in the two-unit case is lower than the price in the one-unit case, and the price difference indicates the difference in the probability that a high value buyer is rationed at the last minute in different cases. If a high-value buyer enters the market early and buys a unit immediately, the seller can sell it at a relatively high price and earn a higher profit than he could earn from running fire sales. However, if no such buyer ever shows up, then the time will eventually come when selling one unit via a fire sale and then following the one-unit pricing strategy is more profitable to the seller. To see the intuition, consider the seller's benefit and cost of liquidating the first unit via a fire sale. The *benefit* is that, by reducing one unit of stock, the seller can charge the high-value buyer who arrives next a higher price for his last unit. On the other hand, the (opportunity) *cost* is that, if more than one high-value buyer arrives before the deadline, the seller cannot serve the second one, who is willing to pay a price higher than the fire sale price. Since a new high-value buyer arrives independently, as the deadline approaches, the probability that more than one high-value buyer arrives before the deadline goes to zero much faster than the probability that one high-value buyer arrives. Thus, the opportunity cost is negligible compared to the benefit, and therefore, the seller has the incentive to liquidate the first unit via a fire sale.

Analyzing a dynamic pricing game with private arrivals is complicated for the following reason. Since the seller can choose both the price and quantity available at any time, he may want to sell his inventory one-by-one. Thus, some buyers may be rationed when demand is less than supply before the game ends. Suppose a buyer was rationed at time t and the seller still holds unsold units. The rationed buyer privately learns that de-

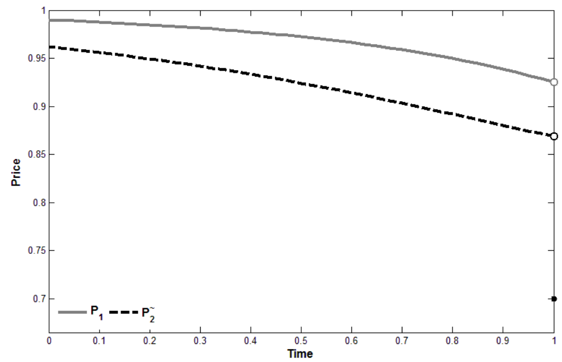

Figure 1: Necessity of Fire Sales Before the Deadline in the Two-Unit Case. The solid (dashed) line shows how the list price will change in the case of one (two) unit of initial stock if low-value buyers are served at the deadline only.

mand is greater than supply at time t and uses the information to update her belief about the number of remaining buyers. Buyers who arrive after this transaction have no such information. As a result, belief heterogeneity among buyers naturally occurs based on their private histories, and buyers' strategies may depend on their private beliefs non-trivially. Such belief heterogeneity evolves over time and becomes more complicated as transactions happen one after another, making the problem intractable.

To overcome this technical challenge, we assume that buyers face inattention frictions. That is, in each "period" with a positive measure of time, instead of assuming that buyers can observe offers all the time, we assume that each buyer notices the seller's offer and makes her purchase decision at her attention times only. In each "period," a buyer independently draws one attention time from an atomless distribution.[5] In addition, buyers' attention can be attracted by an offer with sufficiently low price, that is, a fire sale.[6] This implies that (1) at any particular time, the probability that a buyer observes a non fire sale offer is zero, (2) the probability that more than one buyer observes a non-fire-sale offer at the same time is zero too, and (3)

[5] In the airline ticket example, it is natural to assume each buyer checks the price once or twice per day instead of looking at the airfare website all the time.

[6] In practice, this extra chance is justified by consumers' attention being attracted by advertisements of deals sent by a third party: low price alert e-mails from intermediate websites that offer airfares such as http://www.orbitz.com and http://www.faredetective.com.

all buyers observe a fire-sale-offer when it is posted. As a result, high-value buyers would not be rationed except at deal time. Furthermore, we focus on equilibria where high-value buyers make their purchases upon arrivals. Therefore, a high-value buyer being rationed at deal time attributes failure of her purchase to the competition with low-buyer buyers instead of other high-value buyers, so she cannot infer extra information about the number of buyers in the market. As we will show, there is an equilibrium in which buyers' strategies do not depend on their private histories.[7]

As we described earlier, we are interested in the environment where the seller finds selling only at the deadline and serving only high-value buyers to be suboptimal. In the presence of inattention frictions, the seller cannot guarantee that the high-value buyers will be available at the deadline. Hence, at the deadline, to maximize his profit, the seller has to post a last-minute deal to draw full attention of the market, which naturally leads the seller to start selling early.[8]

We believe that the importance of revenue management studies without commitment is at least threefold. First, in the literature, reputation concerns are commonly cited as a justification of the perfect commitment power of sellers. However, for such a reputation mechanism to work and to act as a legitimate defense of commitment, one needs to understand the benefit and cost of sustaining the commitment price path. Obviously, an in-depth understanding of a world without commitment must be the basis for building the cost of the seller's deviation. Second, studying a model without commitment can help us to evaluate how crucial the perfect commitment assumption is and to what degree the insights we have gained depend on it. Last, a non-commitment model should be the starting benchmark to understand the role of certain selling strategies with the feature of price commitment in reality. For instance, in both the airline and the hotel industries, sellers use the best price guarantee or best available rate policy. That is, if the buyer finds a cheaper price than what he paid within a certain

[7] The idea that, in a continuous-time environment, decision times arrive randomly is not new. See for example, Perry and Reny (1993) and Ambrus and Lu (2010) in bargaining models, and Kamada and Kandori (2011) in revision games. In macroeconomics, there is a large literature analyzing the role of inertia information on sticky prices. See the text-book treatment by Veldkamp (2011). However, none of those papers employ such an assumption to avoid the complexities of private beliefs.

[8] Notice that our economic prediction on the price path does not depend on the presence of inattention frictions. As we mentioned before, a low arrival rate of buyers or the disappearance of present buyers can also exclude the trivial case where the seller is willing to sell at the deadline only. We explore the possibility of disappearing buyers in the extension.

time period, the seller commits to refund the difference and gives the buyer some extra compensation. In a perfect commitment model, it is hard to see the role of these selling policies.

1.1 The Literature

This paper is closely related to two streams of the literature. First, there is a large *revenue management* literature that has examined the market with sellers who need to sell finitely many goods before a deadline and impatient buyers who arrive sequentially.[9] However, as argued by Besanko and Winston (1990), mistakenly treating forward-looking customers as myopic may have an important impact on sellers' revenue. Board and Skrzypacz (2010) characterize the revenue-maximizing mechanism in a model where agents arrive in the market over time. In the continuous time limit, the revenue-maximizing mechanism is implemented via a price-posting mechanism, with an auction for the last unit at the deadline.

In the works mentioned above, perfect commitment of the seller is typically assumed. Little has been done to discuss the case in which a monopolist with scarce supply and no commitment power sells to forward-looking customers. Aviv and Pazgal (2008) consider a two-period case, and so do Jerath, Netessine, and Veeraraghavan (2010). Deb and Said (2012) study a two-period problem where a seller faces buyers who arrive in each period. They show that the seller's optimal contract pools low-value buyers, separates high-value ones, and induces intermediate ones to delay their purchase.

To the best of our knowledge, Chen (2012) and Hörner and Samuelson (2011) have made the first attempt to address the non-commitment issue in a revenue management environment using a multiple-period game-theoretic model. They assume that the seller faces a fixed number of buyers who strategically time their purchases. They show that the seller either replicates a Dutch auction or posts unacceptable prices up to the very end and charges a static monopoly price at the deadline. However, as argued by McAfee and te Velde (2008), arrival of new buyers seems to be an important driving force of many observed phenomena in a dynamic environment. As we will show, the sequential arrival of buyers plays a critical role in the seller's optimal pricing and fire sale decision.

Additionally, our model is also related to the *durable goods* literature in which the seller without capacity constraint sells durable goods to strategic buyers over an infinite horizon. As Hörner and Samuelson (2011) show, the

[9] See the book by Talluri and van Ryzin (2004).

deadline endows the seller with considerable commitment power, and the scarcity of the good changes the issues surrounding price discrimination, with the impetus for buying early at a high price now arising out of the fear that another buyer will snatch the good in the meantime. In the standard durable goods literature, the number of buyers is fixed. However, some papers consider the arrival of new buyers. Conlisk, Gerstner and Sobel (1984) allows a new cohort of buyers with binary valuation to enter the market in each period and show that the seller will vary the price over time. In most periods, he charges a price just to sell immediately to high-value buyers. Periodically, he charges a sales price to sell to accumulated low-value buyers.

In contrast to most durable goods papers, Garrett (2011) assumes that a seller with full commitment power faces a representative buyer who arrives at a random time. Once the buyer arrives, her valuation changes over time. He shows that the optimal price path involves fluctuations over time. Similar to Conlisk, Gerstner and Sobel (1984), most of the time, the seller charges a price just to sell immediately to the arrived buyer when her valuation is high. No transaction implies that either (1) the buyer did not arrive, or (2) she arrived but her valuation is low. After a long time with no transactions, the seller is more and more convinced that the latter is true. As a result, he charges a price acceptable to the arrived buyer with low valuation. Even though, similar to both Conlisk, Gerstner and Sobel (1984) and Garrett (2011), new arrivals and heterogeneous valuation are also the driving force of fire sales in our model, the economic channels are very different. In their papers, the seller has discounting a cost, so charges low price to sell to accumulated low-value buyers in order to reap some profit and avoid delay costs. However, in our model, the seller does not discount and can ensure a unit profit as the fire sales income at the deadline for all inventory. Since the buyers face scarcity, the seller liquidates some goods to convince future buyers to accept higher prices.

The rest of this paper is organized as follows. In Section 2, we present the model setting and define the solution concept we are going to use. In Section 3, we derive an equilibrium in the single-unit case. In Section 4, the multi-units case is studied. In Section 5, we discuss some modelling choices, applications and possible extensions of the baseline model. Section 6 concludes. In Appendix A, we discuss the set of admissible strategies and the solution concept in this game. All proofs are in Appendix B.

2 Model

Environment. We consider a dynamic pricing game between a single seller who has K identical and indivisible items for sale and many buyers. Goods are consumed at a fixed time that we normalize to 1, and deliver zero value after. Time is continuous. The seller has the interval $[0,1]$ of time in which to trade with buyers. There is a parameter Δ such that $1/\Delta \in \mathbb{N}$. The time interval $[0,1]$ is divided into periods: $[0,\Delta)$,$[\Delta, 2\Delta)$, ...$[1-\Delta, 1]$. The seller and the buyers *do not* discount.

Seller. The seller can adjust the price and supply at each moment: at time t, the seller posts the price $P(t) \in \mathbb{R}$, and capacity control $Q(t) \in \{1, 2, ..K(t)\}$, where $K(t) \in \mathbb{N}$ represents the amount of goods remaining at time t, and $K(0) = K$.[10] The seller has a zero reservation value on each item, so his payoff is the summation of all transaction prices.

Buyers. There are two kinds of buyers: low-value buyers (L-buyers, henceforth) and high-value buyers (H-buyers, henceforth). Each buyer has a single unit of demand. Let v_L denote an L-buyer's reservation value of the unit, and v_H that of an H-buyer, where $v_H > v_L > 0$. A buyer who buys an item at price p gets payoff $v - p$ where $v \in \{v_L, v_H\}$.

Population Dynamics. The population structure of buyers changes differently over time. At the beginning, there is no H-buyer in the market. As time goes on, H-buyers arrive privately at a constant rate $\lambda > 0$. Let $N(t)$ be the number of H-buyers. An H-buyer leaves only if her demand is satisfied.[11] For tractability, we assume that the population structure of L-buyers is relatively predictable and stationary. At the beginning of each period, M L-buyers arrive in the market, where $M \in \mathbb{N}$ is common knowledge. When an L-buyer's demand is not satisfied, she leaves the market, and at the end of each period, all L-buyers leave.[12] We assume $M \geq K(0)$.

Transaction Mechanism. If the amount of demand at price $P(t)$ is less than or equal to $Q(t)$, all demands are satisfied; otherwise, $Q(t)$ randomly selected buyers are able to make purchases, and the rest are rationed. A price lower than v_L is always dominated by v_L. Thus, L-buyers do not face non-trivial purchase time decisions. To save notation, we assume that

[10] We assume $Q(t) \neq 0$. However, the seller can post a price sufficiently high to block any transactions.

[11] Our results continue to hold when H-buyers leave the market at a rate $\rho \geq 0$.

[12] An added value of this assumption is that it allows us to highlight our channel to generate fire sales. In Conlisk, Gerstner and Sobel (1984), the presence of periodic sales is driven by the arrival and accumulation of low-value buyers. By assuming that the population structure of low-value buyers is stationary, their classical explanation of a price cycle does not work in our model.

they are non-strategic and will accept any price no higher than v_L. We define such a price as a *deal*.

Definition 1. *A* ***deal*** *is an offer with* $P(t) \leq v_L$.

If $i \leq Q(t)$ goods are sold at time t, the seller's inventory goes down. In other words, $\lim_{t' \searrow t} K(t') = K(t) - i$. Over time, as buyers make purchases, the inventory decreases. Hence, $K : t \to \mathbb{N}$ is a left continuous and non-increasing function. Once $K(t)$ hits zero or time reaches the deadline, the game ends.

Inattention Frictions. We assume that buyers, regardless of their reservation value and arrival times, face inattention frictions. At the beginning of each period, all buyers, regardless of their value, randomly draw an attention time τ, which is uniformly distributed in the time interval of the current period.[13] For an H-buyer who arrives in the period, her attention time in the current period is her arrival time. In the period where the seller posts a *deal* at time τ, each buyer has an additional attention time at time τ in the current period. In the rest of this paper, we call these random attention times exogenously assigned by Nature *regular attention times*, while we call the additional attention time *deal attention times*. A buyer observes the offer posted, $P(t), Q(t)$ and the seller's inventory size, $K(t)$ at her attention time *only*. At that time, she can decide to accept or reject the offer. Rejection is not observed by the seller and other buyers. Since, without deal announcements, each buyer draws her attention time independently, once a buyer observes and decides to take an available offer $P(t) > v_L$, she will not be rationed. Thus the competition among buyers is always intertemporal when $P(t) > v_L$. At deal times when $P(t) \leq v_L$, buyers observe the offer at the same time, so there is direct competition among buyers. Notice that Δ capture the inattention fictions of buyers, and we focus on the case where Δ is small.

History. A non-trivial *seller history* at time t, $h_S^t = (P(\tau), Q(\tau), K(\tau))_{0 \leq \tau < t}$, is a history such that the game is not over before t and it summarizes all relevant transactions and information about offers in the past. Let $\mathcal{H}_S$ be the set of all seller's history. The seller's strategy σ_S determines a price $P(t)$ and capacity control $Q(t)$ given a seller history h_S^t. Due to the buyers' inattention frictions, at any time before the deadline, the seller believes that more than one buyer notices an offer with probability zero. As a result, we focus on the seller's strategy space in which $Q(t) = 1$ for $P(t) > v_L$ without loss of generality.

[13] Our results hold for any atomless distribution with full support.

Let $a(t)$ be an index function such that it is 1 at an H-buyer's attention times, and 0 otherwise. Thus, $a^t = \{a(\tau)\}_{\tau=0}^{t}$ records the history of an H-buyer's past attention times up to t. A non-trivial *buyer history*, $h_B^t = \left\{a^t, \{P(\tau), Q(\tau), K(\tau)\}_{\tau: a(\tau)=1 \text{ and } \tau\in[0,t]}\right\}$. In other words, a buyer remembers the prices, capacity and inventory size she observed at her past attention times. Let $\mathcal{H}_B$ denote the set of all history of an H-buyer. Following Chen (2012) and Hörner and Samuelson (2011), we focus on symmetric equilibria in which an H-buyer's strategy depends only on her history not on her identity. That is to say, the H-buyer's strategy σ_B determines the probability that she will accept the current price $P(t)$ given a buyer's history h_B^t. We focus on a pure strategy profile, so $\sigma_B \in \{0, 1\}$.

2.1 On Continuous Time Games

We choose a continuous time model in this project, since it has technical advantages in answering our questions. Specifically, the determination of the optimal timing for fire sales is in fact an optimal stopping time problem; therefore, the continuous-time properties of this problem make the analysis easier.

However, continuous time raises obstacles to the analysis of dynamic games. First, it is well known that, in a continuous time game, a well-defined strategy may not induce a well-defined outcome. This is analyzed by Simon and Stinchcombe (1989) and Bergin and MacLeod (1993). The reason is that there is no well-defined "last" or "next" period in a continuous time game; hence, players' actions at time t may depend on information arriving instantaneously before t. For example, in our model, one seemingly possible pricing strategy is that the seller sets $P(t) = 10$ if $t = 0$ or $P(s) = 10$ for $s \in [0, t)$; otherwise, $P(t) = 1$. Intuitively, this strategy should imply a price outcome $P(t) = 10$ for any $t \in [0, 1]$. However, any for $t^* \in (0, 1)$, an outcome $P(t) = 10$ for $t \in [0, t^*]$ and $P(t) = 1$ when $t \in (t^*, 1]$ is compatible with the strategy above. See Simon and Stinchcombe (1989) for more examples.

Therefore, to make this game well-defined, we must impose additional restrictions on the set of strategies. Following Bergin and MacLeod (1993), we restrict the seller's choices in the admissible strategy space. The formal restriction is presented in Appendix A, and we provide the intuition here. To construct the set of admissible strategies, we first restrict the strategy to the inertia strategy space. Intuitively speaking, an inertia strategy is such that instead of an instantaneous response, a player can change her decision only after a very short time lag; hence, such strategy cannot be conditional on very recent information. The set of all inertia strategies includes strategies with arbitrarily short lags, so it may not be complete. To capture the instantaneous response of players, we complete the set and use the completion as the feasible strategy set of our game. For each instantaneous response strategy, we identify its associated outcome as follows. First, we find a sequence of inertia strategies converging to the instantaneous strategy. In such a sequence, each inertia strategy has a well-defined outcome, which gives us a sequence of outcomes. Second, we identify the limit of the outcome sequence as the outcome of this instantaneous response strategy. Let Σ_S^* as the admissible strategy space of the seller. Since H-buyers face inattention frictions, they cannot revise their decision instantaneously, so we do not need to impose any restriction on their strategy; let Σ_B^* denote the set of strategies of H-buyers, and let $\Sigma^* = \Sigma_S^* \times \Sigma_B^*$ be the strategy space we study.

2.2 Payoff and Solution Concept

In general, a player's strategy depends on his or her private history. A perfect Bayesian equilibrium in our game is a strategy profile of the seller and the buyers, such that given other players' strategy, each player has no incentive to deviate, and players update their belief via Bayes' rule where possible. However, the set of all perfect Bayesian equilibria of this game is hard to characterize.

We instead look for simple but intuitive *no-waiting equilibria* that satisfy the following properties. First, the equilibrium strategy profile must be simple; that is, players' equilibrium strategies depend on their histories only through the state variables specified later. Second, on the path of play, H-buyers make their purchases once they arrive. Third, we impose a restriction on buyers' beliefs about the underlying history off the path of play: each H-buyer believes that there are no other previous H-buyers presently in the market.

Note that some H-buyers may wait because of the deviation of the seller: the seller can post an unacceptable price for a time interval of positive measure in which H-buyers have to wait for future offers. However, each buyer can observe only finitely many offers at her past attention times and, for the rest of time, she has to form a belief about the underlying history. The perfect Bayesian equilibrium concept does not impose any restriction on those beliefs where the Bayes' rule does not apply. To support a no-waiting equilibrium, we assume that each H-buyer believes that no other H-buyers are waiting in the market. The justification of this refinement can be found in Appendix A.

2.2.1 Payoff

To define the equilibrium, we need to specify an H-buyer's payoff given she believes that no previous H-buyers are waiting in the market. Given a seller's continuation strategy $\tilde{\sigma}_S \in \Sigma_S^*$, other H-buyers' symmetric continuation strategy $\tilde{\sigma}_B \in \Sigma_B^*$, and a buyer's history h_B^t, an H-buyer's payoff from choosing a strategy $\tilde{\sigma}'_B \in \Sigma_B^*$ at her attention time is defined as

$$U\left(\tilde{\sigma}'_B, \tilde{\sigma}_B, \tilde{\sigma}_S, h_B^t\right) = \mathbb{E}_{\tau|t}\left[v_H - P\left(\tau\right)\right]$$

where $\tau \in [t, 1] \cup \{2\}$ is H-buyers' transaction time which is random and depends on the other players' strategies and the population dynamics of buyers. When $\tau = 2$, the buyer does not obtain the good because the seller's stock is sold out before she decides to place an order. In this case, $P(2) = v_H$. At time t, an H-buyer employs a cutoff strategy where she accepts a price if it is less than or equal to some reservation price p, and this reservation price is pinned down by the buyer's indifference condition:

$$v_H - p = \mathbb{E}_{\tau|t}\left[v_H - P\left(\tau\right)\right].$$

Suppose all H-buyers play a symmetric $\tilde{\sigma}_B \in \Sigma_B^*$. The payoff to the seller with stock k from a strategy $\tilde{\sigma}_S \in \Sigma_S^*$ is given by

$$\Pi_k\left(\tilde{\sigma}_B, \tilde{\sigma}_S, h_S^t\right) = \mathbb{E}_\tau\left[P\left(\tau\right) + \Pi_{k-1}\left(\tilde{\sigma}_B, \tilde{\sigma}_S, h_S^\tau\right)\right],$$

where h_S^t is the seller's history, $\Pi_0 = 0$. Because buyers face inattention frictions, by posting any price $P\left(1\right) > v_L$, the seller expects no buyer notices the offer, and his expected profit is zero; by posting a deal price, the seller can sell all of his inventory. Hence, we have

$$\Pi_k\left(\tilde{\sigma}_B, \tilde{\sigma}_S, h_S^1\right) = \begin{cases} 0, & \text{if } P\left(1\right) > v_L, \\ kv_L, & \text{otherwise,} \end{cases}$$

Note that the seller may or may not believe that there are previously arrived H-buyers waiting in the market. His belief about the number of H-buyers depends on the price he posted before.

2.2.2 (No-Waiting) Markov Perfect Equilibrium

We focus on Markov equilibria where an H-buyer makes her purchase decision based on two state variables: calendar time and inventory size, and she makes the purchase on her arrival time on the equilibrium path. The seller's equilibrium strategy depends on calendar time, inventory size, and his estimated number of present H-buyers. Specifically, based on the his realized history, the seller forms a belief about the number of H-buyers, $N(t)$. Let $\Phi(t)$ be the seller's belief over $N(t)$ where $\Phi_n(t)$ represents the probability that the seller believes that $N(t) = n$. Furthermore, the seller needs to distinguish between H-buyers whose attention times were before t, and those whose attention times are equal to or after t in the current period. Let $N^-(t)$ denote the number of H-buyers whose attention times were before t, and let $N^+(t)$ denote those whose attention times are equal to or after t in the current period. Let $\Phi^-(t)$ and $\Phi^+(t)$ be the seller's beliefs over $N^-(t)$ and $N^+(t)$, where $\Phi_n^-(t)$ (and $\Phi_n^+(t)$) represent that the seller believes that $N^-(t) = n$ (and $N^+(t) = n$) at time t. Given $\Phi^-(t)$ and $\Phi^+(t)$, we can calculate the seller's belief as follows: for any $n \in \mathbb{N}$, $\Phi_n(t) = \sum_{i=0}^{n} \Phi_i^-(t)\, \Phi_{n-i}^+(t)$.

Definition 2. *The set* $\Xi_S \subset [0,1]^\infty$ *is a collection of seller's beliefs* $[\Phi^-(t), \Phi^+(t)]$ *such that can be reached after any seller history.*

As we mentioned before, we restrict the strategy space such that $Q(t) = 1$ for $P(t) > v_L$. Hence, the seller only needs to choose the price. We define a Markovian strategy profile as follows.

Definition 3. *A strategy profile* (σ_S, σ_B) *is Markovian if and only if*

1. *the seller's strategy* σ_S *depends on the seller's history via* $(t, K(t), \Phi^-(t), \Phi^+(t))$ *only, and*

2. *the H-buyer's strategy* σ_B *depends on the buyer's history via* $(t, K(t))$ *only.*

In the definition, the H-buyer's strategy is a function of the calendar time and the seller's inventory size, but it does not imply that the number of other H-buyers is payoff irrelevant to an H-buyer. In fact, an H-buyer's continuation value does depend on her belief about the number of other H-buyers. However, we focus on no-waiting equilibria where each H-buyer believes that no other H-buyer is waiting in the market; thus, her strategy does not depend on her belief about the number of other H-buyers non-trivially.

Furthermore, we can define the solution concept in this game.

Definition 4. *A (no-waiting) Markov perfect equilibrium (henceforth equilibrium) consists of a (pure) strategy profile* (σ_B^*, σ_S^*) *such that, for any seller's history* h_S^t*, and for any buyer's history* h_B^t*,*

1. *given the seller's strategy* σ_S^**, other buyers' strategy* σ_B^**,*
$$U\left(\sigma_B^*, \sigma_B^*, \sigma_S^*, h_B^t\right) \geq U\left(\tilde{\sigma}_B, \sigma_B^*, \sigma_S^*, h_B^t\right)$$
for any admissible $\tilde{\sigma}_B$*,*

2. *given buyers' strategy* σ_B^**,*
$$\Pi_k\left(\sigma_B^*, \sigma_S^*, h_S^t\right) \geq \Pi_k\left(\sigma_B^*, \tilde{\sigma}_S, h_S^t\right)$$
for any admissible $\tilde{\sigma}_S$*,* $k \in \{1, 2, ..K\}$*,*

3. *the seller's belief is consistent with the seller's history and* (σ_B^*, σ_S) *for any admissible strategy* $\sigma_S \in \Sigma_S^*$*, and*

4. (σ_B^*, σ_S^*) *is Markovian.*

Nonetheless, note that potential deviations strategy can be either Markovian or non-Markovian.

Over time, the seller's belief evolves based on the realized history. We leave the formal law of motion of $\Phi^+(t)$ and $\Phi^-(t)$ to the Appendix B but provide some intuitive description here. The seller's belief updating is driven by four forces. First, at any time t, there are exogenous arrivals. When the price is too high to be accepted by newly arrived H-buyers, they have to wait and therefore $N^-(t)$ increases. Second, since each H-buyer independently draws her attention time, in a small but non-trivial time interval, an H-buyer, if she is in the market and her attention time in the current period does not pass, observes the offer posted with positive probability. As a result, if an equilibrium offer is posted but the time without transactions grows, H-buyers are likely to be fewer, and therefore, the seller adjusts his belief about $N^+(t)$. Alternatively, if the offer posted is not acceptable to H-buyers, the seller believes that some H-buyers may have observed but rejected it, so $N^-(t)$ increases but $N^+(t)$ decreases. Third, as time goes to the end of the period, all buyers' attention time passes, so $N^+(t)$ converges to zero, and $N^-(t)$ converges to $N(t)$. At the beginning of each period, all remaining buyers can draw a new attention time within the current period, so $N^+(t^+) = N^-(t^-)$ when $t = l\Delta$ for $l = 0, 1, 2, ...1/\Delta - 1$. Last, the seller's belief jumps after each transaction because of the endogenous departure of buyers. The first two forces make the seller's belief smoothly update, but the last two make it jump.

Notice that in many dynamic price discrimination games, the seller's equilibrium pricing strategy is history dependent rather than Markovian, which makes the problem less tractable. In a two-period model, Fudenberg and Tirole (1983) show that there is no Markov equilibria. The non-existence of Markov equilibria continues to hold in an infinite horizon dynamic pricing game. See the discussion by Gul, Sonnenschein and Wilson (1986) in a durable goods environment. The reason is that if a buyer rejects an offer at a particular time, the continuation belief about the buyer's type would change dramatically. In our model, thanks to the presence of inattention frictions, the seller cannot infer any information if a particular offer is not accepted, since the probability the offer was observed by a buyer is zero. As we will show, there is a Markov equilibrium.

3 Single Unit

We start by analyzing the game where $K(0) = 1$, the seller has one unit to sell. Deriving equilibria in this game is the first step forward the analysis of more general games. We first provide an intuitive conjecture on an equilibrium of this game and verify our conjecture. Furthermore, we show that the equilibrium we proposed is the unique equilibrium.

The first observation is that the seller can ensure a profit v_L because there are M L-buyers at the deadline. An intuitive conjecture of the seller's strategy is to serve the H-buyers only before the deadline to obtain a profit higher than v_L and charge v_L at the deadline if no H-buyer arrives. Since an H-buyer would like to avoid a competition with (1) L-buyers at the deadline, and (2) other H-buyers who may arrive before the deadline, she is willing to forgo some surplus and accept a price higher than v_L. Moreover, as deadline approaches, the competition coming from newly arrived H-buyers becomes less and less intense, and therefore the H-buyer's *reservation price* declines.

Specifically, we conjecture that in equilibrium, the seller charges a price such that: (1) H-buyers accept it on arrivals, and (2) low type buyers make their purchases only at the deadline if the good is still available. The optimality of the seller's pricing rule implies that, before the deadline, an H-buyer is indifferent between purchasing at time t and waiting: on the one hand, if the H-buyer strictly prefers to purchase the good immediately, the seller can raise the price a little bit to increase his profit; on the other hand, if the price is so high that the H-buyer strictly prefers to wait, the transaction will not happen at time t and all H-buyers wait in the market. Furthermore, we will show that accumulating H-buyers is suboptimal for the seller because the H-buyers' reserve prices are declining over time. At the deadline, the seller will charge the price v_L to clean out his stock since he believes that there are no H-buyers left.

We give a heuristic description of the equilibrium in the main text and leave the formal analysis to the Appendix B. At the deadline, the H-buyer's reservation price is v_H. However, the probability that an H-buyer's regular attention time is at the deadline is zero; thus, the dominant pricing strategy for the seller is to post a deal price v_L to obtain a positive profit. As a result, in any equilibrium, $P(1) = v_L$. For the rest of the time, we denote $p_1(t)$ as an H-buyer's reservation price at her attention time $t < 1$ and the inventory size $K(t) = 1$. Consider an H-buyer with an attention time $t \in [1-\Delta, 1)$; thus, the probability that new H-buyers arrive before the deadline is $1 - e^{-\lambda(1-t)}$. Suppose this H-buyer understands that on the path of play, no H-buyer who has arrived before her waited. Therefore, she believes that she is the only H-buyer in the market. She then faces the following trade-off:

1. if she accepts the current offer, she gets the good for sure at a price which is higher than v_L;
2. if she does not accept the current offer, the seller will believe that no H-buyer arrived and to obtain a positive profit, he will charge a price v_L to liquidate the good at the deadline. In the latter situation, the H-buyer has to compete with M L-buyers for the item, and the probability she is not rationed is $\frac{1}{M+1}$.

These considerations can pin down an H-buyer's reservation price, $p_1(t)$, at which she is indifferent between accepting the offer or not at time t. Specifically, the indifference condition of an H-buyer whose attention time is t is given as follows:

$$v_H - p_1(t) = e^{-\lambda(1-t)} \frac{1}{M+1} (v_H - v_L). \tag{1}$$

The left-hand side represents the H-buyer's payoff if she purchases the good now; the right-hand side represents the expected payoff if she waits, which is risky because (1) other H-buyers may arrive in $(t, 1)$ with a probability $1-e^{-\lambda(1-t)}$, and (2) she has to compete with M L-buyers at the deadline. Differentiating equation (1) with respect to t, we have $\dot{p}_1(t) = -\lambda [v_H - p_1(t)]$.

Letting $t \to 1$, we obtain the limit price right before the deadline,

$$p_1\left(1^-\right) = \frac{M}{M+1} v_H + \frac{1}{M+1} v_L. \tag{2}$$

Hence, if M is large, the limit price right before the deadline is very close to v_H. Note that $p_1\left(1^-\right)$ is different from the H-buyer's actual reservation price at the deadline, v_L. Let $U_{1-\Delta}$ denote an H-buyer's expected utility at the beginning of the last period. Since her attention time, $\tilde{t}$, is a random variable, we have

$$\begin{aligned} U_{1-\Delta} &= \int_{1-\Delta}^{1} \frac{1}{\Delta} e^{-\lambda\left(\tilde{t}-1+\Delta\right)} \left[v_H - p_1\left(\tilde{t}\right)\right] d\tilde{t} \\ &= \int_{1-\Delta}^{1} \frac{1}{\Delta} \left[e^{-\lambda\Delta} \frac{v_H - v_L}{M+1}\right] d\tilde{t}. \end{aligned} \tag{3}$$

Notice that, for each $\tilde{t}$, the H-buyer's ex ante payoff, by considering the risk of the arrival of new buyers and the price declining until $\tilde{t}$, is $e^{-\lambda\Delta} \frac{v_H - v_L}{M+1}$, which implies that an H-buyer at the beginning of the last period, is indifferent between being assigned any attention time in the current period. Hence, $U_{1-\Delta} = v_H - p_1\left(1-\Delta\right)$.

Now, consider the H-buyer's reservation price at an earlier time. Note that, when $K\left(0\right) = 1$, the seller can ensure a profit v_L at any time by charging the fire sale price. However, he expects to charge a higher price to H-buyers who arrive early and want to avoid competition with H-buyers who arrive in the future and L-buyers. As a result, the fire sale price v_L is charged only at the deadline. At any other time t, the seller targets H-buyers only and offers a price $p_1\left(t\right)$. Consider an H-buyer whose attention time is $t \in [1-2\Delta, 1-\Delta)$. Her indifference condition is given by

$$v_H - p_1\left(t\right) = e^{-\lambda(1-\Delta-t)} U_{1-\Delta}. \tag{4}$$

where the left-hand side represents the H-buyer's payoff if she purchases the good now; the right-hand side represents the expected payoff if she waits, with probability $e^{-\lambda(1-\Delta-t)}$, she is still in the market at the beginning of the next period and the good is still available; so she can draw a new attention time in the last period and expect a payoff $U_{1-\Delta}$. Differentiating equation (4) with respect to t, we have $\dot{p}_1\left(t\right) = -\lambda\left[v_H - p_1\left(t\right)\right]$. As t goes to $1-\Delta$, $v_H - p_1\left(t\right)$ converges to $U_{1-\Delta}$. As a result, $p_1\left(t\right)$ is differentiable in $[1-2\Delta, 1)$. Repeating the argument above for $1/\Delta$ times, we have the ordinary differential equation (ODE, henceforth) for the H-buyers' reservation price

$p_1(t)$ such that

$$\dot{p}_1(t) = -\lambda(v_H - p_1(t)) \text{ for } t \in [0,1), \tag{5}$$

with a boundary condition (2). In our conjectured equilibrium, the price the seller charges is $p_1(t)$ for $t \in [0,1)$ and it jumps down to v_L at the deadline.

Similarly, we can derive the seller's payoff $\Pi_1(t)$. At the deadline, $\Pi_1(1) = v_L$ since the good is sold for sure at the fire sale price. Before the deadline, for a small $dt > 0$, the profit follows the following recursive equation:

$$\begin{aligned} \Pi_1(t) &= p_1(t)\lambda dt + (1-\lambda dt)\,\Pi_1(t+dt) + o(dt)\,, \\ &= p_1(t)\lambda dt + (1-\lambda dt)\left[\Pi_1(t) + \dot{\Pi}_1(t)\,dt\right] + o(dt)\,, \end{aligned}$$

where an H-buyer arrives and purchases the good at time t with probability λdt, and no H-buyer arrives with a complementary probability. By taking $dt \to 0$, the seller's profit must satisfy the following ODE:

$$\dot{\Pi}_1(t) = \lambda\left[\Pi_1(t) - p_1(t)\right], \tag{6}$$

with a boundary condition $\Pi_1(1) = v_L$. Note that, even though the equilibrium price is not continuous in time at the deadline, the seller's profit is because the probability that the transaction happens at a price higher than v_L goes to zero as t approaches the deadline.

In short, in our conjectured equilibrium, H-buyers accept a price not higher than their reservation price $p_1(t)$, and the seller posts such price for any $t < 1$, and v_L at the deadline. No H-buyer waits on the path of play. The next question is whether players have the incentive to follow the conjectured equilibrium strategies. A simple observation is that no H-buyer has the incentive to deviate since she is indifferent between taking and leaving the offer at any attention time. What about the seller? Does the seller have the incentive to do so and accumulate H-buyers for a while before the deadline? The answer is again no. This is because each buyer believes that no previous buyers are waiting in the market, and the seller is going to follow the equilibrium pricing rule in the continuation play. Since the H-buyer's reservation price declines over time, the seller always wants to serve the earliest H-buyer. Hence, the seller's equilibrium expected payoff at t is given by

$$\Pi_1(t) = \int_t^1 e^{-\lambda(s-t)}\lambda p_1(s)\,ds + e^{-\lambda(1-t)}v_L.$$

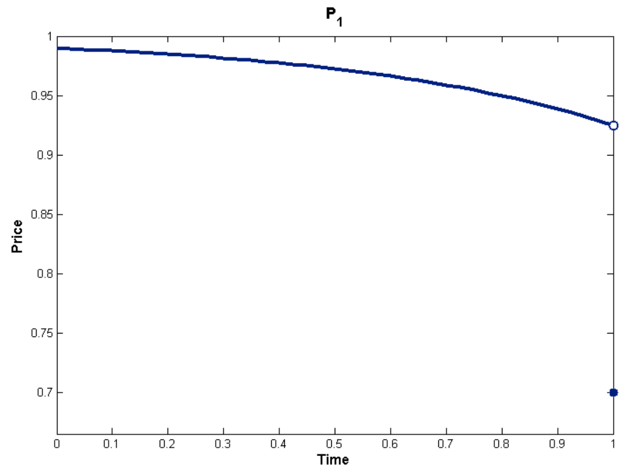

Figure 2: The equilibrium price path in the single-unit case, $K = 1$. The parameter values are $v_H = 1, v_L = 0.7, M = 3$, and $\lambda = 2$.

A simulated equilibrium price path can be found in Figure 2.

Formally,

Proposition 1. *Suppose $K = 1$. There is a unique equilibrium in which,,*

1. *for any non-trivial seller's history, the seller posts a price, $P(t)$ s.t.*

$$P(t) = \begin{cases} p_1(t), & \text{if } t \in [0,1) \\ v_L, & \text{if } t = 1, \end{cases}$$

where

$$p_1(t) = v_H - \frac{v_H - v_L}{M+1} e^{-\lambda(1-t)},$$

2. *an H-buyer accepts a price at her attention time $t \in [0,1)$ if and only if it is less than or equal to $p_1(t)$ and she accepts any price no higher than v_H at the deadline.*

Notice that neither $p_1(t)$ not $\Pi_1(t)$ depends on Δ because each H-buyer makes her purchase once she arrives but does not draw additional attention time on the path of play.

Fire sales appear with positive probability at the deadline only, that is, the last-minute deal. With probability $e^{-\lambda}$, no H-buyer arrives in the market and the seller posts the last-minute deal. The good is not allocated to an L-buyer unless no H-buyer arrives. As a result, the allocation rule is efficient.

4 Multiple Units

In this section we consider the general case in which the seller has $K > 1$ units to sell. Since most intuition can be explained for the two-unit case, we provide a heuristic description of the equilibrium in a two-unit case, and we then state the equilibrium for $K > 2$.

4.1 The Two-Unit Case

Consider the case where $K = 2$. A simple observation is that, after the first transaction at time τ, $K(t) \leq 1$ for $t \in (\tau, 1]$, and what happens afterwards is characterized by Proposition 1. The question is how the first transaction happens: what is the sale price and when does the H-buyer accept the offer? Note that the seller always has a choice to post a price v_L at any t. Since this price is so low that L-buyers can afford it, a transaction will happen for sure and the seller's stock switches to $K(t^+) = K(t) - 1$. In equilibrium, the earliest time at which the seller is willing to sell the first item at the price v_L is denoted by t_1^*. In principle, when $K(t) = 2$, t_1^* can be any time before or at the deadline. As we have shown in Proposition 1, in any continuation game with $K(t) = 1$, on the equilibrium path, the seller charges the price v_L only at the deadline; hence, the last equilibrium fire sale time is always $t_0^* = 1$. However, it is not clear yet when the first equilibrium fire sale time is. Note that, because of the scarcity of the goods at the price v_L, an H-buyer may be rationed at t_1^*. Consequently, she is willing to pay a higher price before t_1^*.

We conjecture that the equilibrium should satisfy the following properties. Before t_1^*, the seller posts a price such that an H-buyer is willing to purchase the good once she arrives. Once an H-buyer buys the good, the amount of stock held by the seller jumps to one. From that moment on, the equilibrium is described by Proposition 1. Similar to the single-unit case, when $K(t) = 2$, an H-buyer's reservation price at $t \leq t_1^*$, $p_2(t)$, satisfies the following ODE:

$$\dot{p}_2(t) = -\lambda [p_1(t) - p_2(t)] \text{ for } t \in [0, t_1^*) \tag{7}$$

The intuition is as follows. Suppose, at $t < t_1^*$, an H-buyer sees the price $p_2(t)$. It is risky for her to wait because a new H-buyer arrives at rate λ and gets the first good at price $p_2(t)$, in which case the original buyer can get the second good only at price $p_1(t)$. At her attention time t, the H-buyer is indifferent between taking the current offer and waiting only if the price declining effect, measured by $\dot{p}_2(t)$, can compensate the possible loss.

Since the seller may obtain a higher unit-profit by selling a good to an H-buyer instead of to an L-buyer, a reasonable conjecture is as follows. In equilibrium, the seller does not run any fire sales prior to the deadline. In other words, the first fire sale time is $t_1^* = 1$, and the seller's optimal price path, $P(t)$, is such that (1) $P(t) > v_L$ for $t < 1$, (2) an H-buyer takes the offer once she arrives, and (3) the seller runs a clearance sale at the deadline. Now that $K(t) = 2$, the equilibrium price satisfies the ODE (7) with $t_1^* = 1$. At the deadline, the seller has to post v_L, and an H-buyer can obtain a good at the deal price with probability $\frac{2}{M+1}$; thus, the boundary condition of the ODE (7) at $t = 1$ is $p_2(1^-) = \frac{2}{M+1} v_H + \frac{M-1}{M+1} v_L$. This strategy profile, however, is not an equilibrium!

Lemma 1. *In any equilibrium, $t_1^* < 1$.*

Lemma 1 rules out the aforementioned conjecture. To see why, first note that $p_2(t) < p_1(t)$ for $t < 1$ since an H-buyer is more likely to get the good when the supply is 2. As t approaches the deadline, the probability that a new H-buyer arrives before the deadline becomes smaller and smaller. The probability that only one H-buyer arrives before the deadline is approximated by $\lambda(1-t)$. In this case,

1. if the seller naively posts price $p_2(t)$, his profit is $p_2(\tau) + v_L$ where τ is the H-buyer's arrival time.

2. Alternatively, if the seller runs a one-unit fire sale before the arrival, he can ensure a payoff of v_L immediately and expect a price $p_1(\tau) > p_2(\tau)$ in future.

When t is close to the deadline, the *benefit* of price cutting is approximated by $p_1(1) - p_2(1)$. On the other hand, there is an *opportunity cost* to holding a fire sale before the deadline. More than one H-buyer may arrive before the deadline and the probability of this event is approximated by $\lambda^2 (1-t)^2$. In this case, if the seller naively posts price $p_2(t)$ and $p_1(t)$ to the end but does not post v_L, his profit is approximated by $p_2(1) + p_1(1)$. Thus the opportunity cost of the fire sale is approximated by $p_2(1) - v_L$ when t is close to the deadline. As t goes to 1, $\lambda^2 (1-t)^2$ goes to zero at a higher speed than $\lambda(1-t)$; thus, the cost is dominated by the benefit for t

close enough to 1, and therefore, the seller will post the fire sale price v_L to liquidate one unit at $t_1^* < 1$ to raise future H-buyers' reservation price. In other words, the fire sale plays the role of a commitment device.

We leave the formal equilibrium construction to the Appendix B but illustrate the idea here to provide intuition. Suppose Δ is small enough; thus, a buyer can make her next purchase decision soon after one rejection. Suppose buyers believe that the fire sale time is t_1^*. For $t < t_1^*$, and $K(t) = 2$, an H-buyer's reservation price satisfies the ODE (7); for $t \in [t_1^*, 1)$ and $K(t) = 2$, H-buyers believe that the seller is going to post v_L immediately, and thus their reservation prices satisfies the following equation

$$v_H - p_2(t) = \frac{1}{M+1}(v_H - v_L) + \frac{M}{M+1}[v_H - p_1(t)],$$

where the left-hand side of the equation is the H-buyer's payoff by accepting her reservation price and obtaining the good now, and the right-hand side is her expected payoff by rejecting the current offer. With probability $\frac{1}{M+1}$, the H-buyer gets the good at the deal price right after time t, and with a complementary probability, an L-buyer gets the deal and the H-buyer has to take $p_1(t)$ at her next attention time. Since Δ is small, one can ignore the arrivals and the time difference between two adjacent attention times of the H-buyer, and therefore, the H-buyer's reservation price at $t \in [t_1^*, 1)$ is given by

$$p_2(t) = \frac{1}{M+1} v_L + \frac{M}{M+1} p_1(t).$$

The incentive-compatible condition of the H-buyer implies that $p_2(t)$ must be continuous at t_1^*, and thus the boundary condition of the ODE (7) is

$$p_2(t_1^*) = \frac{1}{M+1} v_L + \frac{M}{M+1} p_1(t_1^*). \tag{8}$$

As a result, an H-buyer's reservation price at t when $K(t) = 2$ critically depends on her belief about t_1^*.

Given H-buyers' common beliefs about t_1^*, and their reservation prices when $K(t) = 2$, the seller's problem is to choose his optimal fire sale time to maximize his profit; i.e.:

$$\Pi_2(t) = \max_{t_1} \int_t^{t_1} e^{-\lambda(s-t)} \lambda [p_2(s) + \Pi_1(s)]\, ds + e^{-\lambda(t_1 - t)} [v_L + \Pi_1(t_1)].$$

In equilibrium, buyers' belief is *correct*, so the seller's optimal fire sale time is t_1^* itself. The first-order-condition of the seller's problem at t_1^* is:

$$\lambda [p_2(t_1^*) - v_L] + \dot{\Pi}_1(t_1^*) = 0. \tag{9}$$

At t_1^*, a transaction happens at price v_L for sure, so we have

$$\Pi_2(t_1^*) = \Pi_1(t_1^*) + v_L, \tag{10}$$

which is the well-known *value-matching condition* in an optimal stopping time problem.

For $t < t_1^*$, and $K(t) = 2$, the seller posts the H-buyer's reservation price, $p_2(t)$, and his expected profit is given by

$$\Pi_2(t) = \lambda dt\,[p_2(t) + \Pi_1(t+dt)] + (1-\lambda dt)\,\Pi_2(t+dt) + O\left(dt^2\right).$$

Taking $dt \to 0$, the seller's profit satisfies the following *Hamilton-Jacobi-Bellman* (henceforth, HJB) equation

$$\dot{\Pi}_2(t) = -\lambda\,[p_2(t) + \Pi_1(t) - \Pi_2(t)]. \tag{11}$$

Combining (9), (10) and (11) at t_1^* yields

$$\dot{\Pi}_2(t_1^*) = \dot{\Pi}_1(t_1^*), \tag{12}$$

which is known as the *smooth-pasting condition*.

As a result, at the equilibrium fire sale time t_1^*, three necessary conditions (8), (10), and (12) must hold. The necessity of the value-matching condition (10) and the smooth-pasting condition (12) comes from the optimal stopping time property of the interior fire sale time, and condition (8) results from the H-buyers' incentive-compatible condition. When time is arbitrarily close to t_1^*, the probability that new H-buyers arrive before t_1^* shrinks, and the H-buyer needs to choose between taking the current offer and waiting to compete with the L-buyers for the deal. Therefore, her reservation price must make the H-buyer indifferent between taking it and rejecting it. If t is not close to t_1^*, the competition from newly arrived H-buyers before t_1^* is non-trivial, and therefore, to convince an H-buyer to accept the price, it must satisfy the ODE (7) with a boundary condition (8) at t_1^*. The seller's equilibrium profit when $K(t) = 2$ is given by

$$\Pi_2(t) = \begin{cases} \Pi_1(t) + v_L, & t \geq t_1^* \\ \int_t^{t_1^*} e^{-\lambda(s-t)}\lambda\,[p_2(s) + \Pi_1(s)]\,ds + e^{-\lambda\left(t_1^*-t\right)}\,[v_L + \Pi_1(t_1^*)], & t < t_1^* \end{cases}$$

where t_1^* satisfies conditions (8),(10) and (12), $\Pi_1(t)$ is characterized in Proposition 1, and $p_2(t)$ satisfies ODE (7) with a boundary condition (8).

The following proposition formalizes our heuristic description of the equilibrium.

Proposition 2. *Suppose $K(0) = 2$. There is a $\bar{\Delta} > 0$ such that when $\Delta \in \left(0, \bar{\Delta}\right)$, there exists a unique equilibrium. In this equilibrium, there is a fire sale time $t_1^* \in [0, 1)$ such that:*

1. *on the path of play, the seller posts*

$$P(t) = \begin{cases} p_1(t), & \text{when } t < 1 \text{ and } K(t) = 1, \\ p_2(t), & \text{when } t < t_1^* \text{ and } K(t) = 2, \\ v_L, & \text{otherwise.} \end{cases}$$

where

$$p_2(t) = \begin{cases} v_H - \frac{v_H - v_L}{M+1} e^{-\lambda(1-t)} \left[e^{\lambda\left(1-t_1^*\right)} + \frac{M}{M+1} + \lambda\left(t_1^* - t\right) \right], & t \in [0, t_1^*), \\ \frac{1}{M+1} v_L + \frac{M}{M+1} p_1(t), & t \in [t_1^*, 1), \end{cases}$$

and $p_1(t)$ is specified in Proposition 1,

2. *an H-buyer's reservation price is $p_1(t)$ and $p_2(t)$ when $t < 1$, $K(t) = 1$ and 2, respectively, and v_H at $t = 1$.*

Note that the first fire sale time t_1^* always exists, even though for some parameters it is not an interior solution, i.e., $t_1^* = 0$. In that case, the seller is so pessimistic about the arrival of H-buyers that he prefers to liquidate the first unit at the very beginning. Figure 3 shows a simulated equilibrium price path.

In the equilibrium, for $t < t_1^*$, the price is $p_2(t)$, and it jumps up to $p_1(t)$ once a transaction happens. If there is no transaction before t_1^*, the price jumps down to v_L, and one unit is sold immediately; it then jumps up to the path of $p_1(\cdot)$. The first fire sale actually happens at t_1^* with probability $e^{-\lambda\left(1-t_1^*\right)}$. Since two or more H-buyers arrive after t_1^* with positive probability, the allocation is *inefficient*. However, in contrast to the standard monopoly pricing game where the inefficiency results from the seller's withholding, the inefficiency in this game arises from the scarce good being misallocated to L-buyers when many H-buyers arrive late.

It is worth noting that our equilibrium prediction on the fire sale critically depends on two assumptions: (1) H-buyers are forward-looking, and (2) the number of L-buyers is finite. First, suppose each H-buyer can draw at most one attention time, and thus she cannot strategically time her purchase. As a result, for any $t \in [0, 1]$ and $k \in \mathbb{N}$, the H-buyers' reservation price is always $p_k(t) = v_H$ for any k. Hence, the optimal price path $P(t) = v_H$ when $t < 1$ and $P(t) = v_L$ when $t = 1$ for any $k \in \mathbb{N}$. In this particular model, the

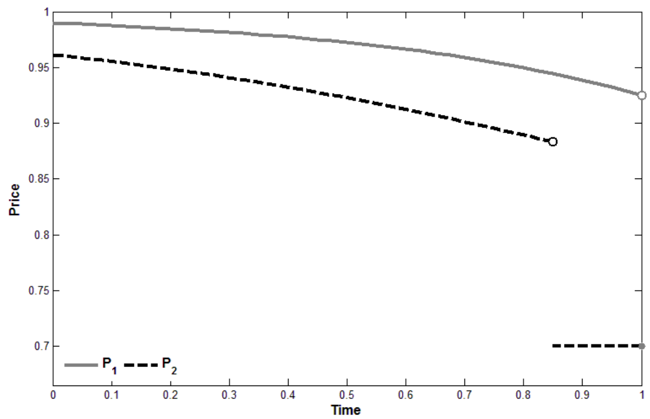

Figure 3: The equilibrium price path for the two-unit case. The solid line is the equilibrium price when $K(t) = 1$, while the dashed line is that when $K(t) = 2$. The first fire sale time is $t_1^* = 0.84$. When $t \geq t_1^*$ and $K(t) = 2$, the seller posts the deal price, v_L, to liquidate the first unit immediately. The parameter values are $v_H = 1, v_L = 0.7, M = 3$, and $\lambda = 2$.

price is constant until $t = 1$. In a more general model, for example, buyers may have a heterogeneous reservation value $v \in [v_L, v_H]$. Talluri and van Ryzin (2004) consider many variations of this model. In these models, the result does not depend on the seller's commitment power. Second, when the number of L-buyers, M, is finite, an H-buyer can get a good at the deal price with positive probability. However, if M is infinity, the probability that an H-buyer can get a good at the deal price is zero. Hence, the difference between $p_1(t)$ and $p_2(t)$ disappears. In fact, an H-buyer cannot expect any positive surplus and is willing to accept a price v_H at any time.

4.2 The General Case

In general, the seller has K units where $K \in \mathbb{N}$. In the equilibrium, the seller may periodically post a deal price before the deadline. Specifically, there is a sequence of fire sale times, $\{t_k^*\}_{k=1}^{K-1}$, such that $t_{k+1}^* \leq t_k^*$ for $k \in \{1, 2, ..K-1\}$. When $t \in [t_1^*, 1)$, if $K(t) = 1$, the seller posts $p_1(t)$; if $K(t) > 1$, the seller liquidates $K(t) - 1$ units via a fire sale immediately and makes his inventory size jump to 1. When $t \in [t_2^*, t_1^*)$, if $K(t) = k$, the seller posts $p_k(t)$ for $k = 1, 2$ and serves H-buyers; if $K(t) > 2$, he liquidates

$K(t)-2$ units via a fire sale. By the same logic, for any $k \in \{2, ...K-1\}$, when $t \in [t_k^*, t_{k-1}^*)$, the seller's equilibrium pricing strategy is as follows: if $K(t) \leq k$, the seller serves H-buyers only by posting a price $P(t) = p_k(t)$; if $K(t) > k$, the seller posts a deal price and liquidates $K(t)-k$ units of stock immediately.

We derive the equilibrium by induction. Suppose in the $K-1$-unit case, H-buyers' reservation price is $p_k(t)$ for $k \in \{1, 2, ..K-1\}$, and the seller's equilibrium strategy is consistent with the description above. The seller's equilibrium profit is represented by $\Pi_k(t)$ for $k \in \{1, 2, ..K-1\}$. Now we construct the H-buyers' reservation price and the seller's pricing strategy and payoff in the K-units case. To satisfy the H-buyers' incentive-compatible condition, the equilibrium price at t when $K(t) = k \in \mathbb{N}$ satisfies the following differential equation:

$$\dot{p}_K(t) = -\lambda \left[p_{K-1}(t) - p_K(t)\right] \text{ for } t \in [0, t_{K-1}^*), \tag{13}$$

where t_{K-1}^* is the first equilibrium fire sale time when $K(t) = K$, and

$$p_K(t) = \frac{i}{M+1} v_L + \frac{M+1-i}{M+1} p_{K-i}(t) \text{ for } t \in [t_{K-i}^*, t_{K-i-1}^*)$$

where $i = 1, 2, ...K-1$ and $t_0^* = 1$. The incentive-compatible condition of the H-buyer implies that $p_K(t)$ must be continuous at t_{K-1}^*; thus, the boundary condition of the ODE (13) is given by $p_K\left(t_{K-1}^*\right) = \frac{1}{M+1} v_L + \frac{M}{M+1} p_{K-1}\left(t_{K-1}^*\right)$, and therefore, the H-buyer's best response is specified for any $t \in [0, 1]$ and $k \in \{1, 2, ...K\}$.

The seller's problem is to choose the optimal fire sale time and quantity to maximize his profit. Formally,

$$\begin{aligned}\Pi_K(t) = \max_{t_{K-1} \in [0,1]} \int_t^{t_{K-1}} & e^{-\lambda(\tau - t)} \lambda \left[p_K(s) + \Pi_{K-1}(s)\right] ds \\ & + e^{-\lambda(t_{K-1} - t)} \left[v_L + \Pi_{K-1}(t_{K-1})\right].\end{aligned}$$

In equilibrium, buyers' beliefs are correct, so the seller's optimal fire sales time when $K(t) = K$ is t_{K-1}^*, which satisfies the value-matching and the smooth-pasting conditions.

If there exists an interior solution, t_K^* is pinned down as follows. At t_{K-1}^*,

$$p_K\left(t_{K-1}^*\right) = \frac{1}{M+1} v_L + \frac{M}{M+1} p_{K-1}\left(t_{K-1}^*\right), \tag{14a}$$

$$\Pi_K\left(t_{K-1}^*\right) = \Pi_{K-1}\left(t_{K-1}^*\right) + v_L, \tag{14b}$$

$$\dot{\Pi}_K\left(t_{K-1}^*\right) = \dot{\Pi}_{K-1}\left(t_{K-1}^*\right). \tag{14c}$$

In equilibrium, we have $t^*_{K-1} \leq t^*_{K-2}$. The intuition is simple. In a no-waiting equilibrium, no previous arrived H-buyers are waiting in the market; thus, the demand from H-buyers shrinks as the deadline approaches. What is more, the probability that more than k H-buyers arrive before the deadline is approximated by $\lambda^k (1-t)^k$ when the current time t is close to the deadline. Apparently, the higher k is, the smaller the probability is. Hence, the seller who holds more units has the incentive to liquidate part of his inventory early. What is more, when Δ is small, on the path of play, the seller does not run more than one fire sale in the same period.

For a history in which $K(t) = k \in \{1, 2, ..K\}$ and $t \in [t_{k'}, t_{k'-1})$ for $k' < k-1$, the seller would try to liquidate multiple units of goods as soon as possible. The seller's profit when $K(t) = k$ is given by

$$\Pi_k(t) = \begin{cases} \{\int_t^{t^*_{k-1}} e^{-\lambda(s-t)} \lambda \left[p_k(\tau) + \Pi_{k-1}(\tau)\right] d\tau & \text{if } t < t^*_{k-1} \\ \quad + e^{-\lambda\left(t^*_{k-1}-t\right)} \left[v_L + \Pi_{k-1}\left(t^*_{k-1}\right)\right]\}, & \\ v_L(k-k') + \Pi_{k'}(t), & \text{if } t \in [t^*_{k'}, t^*_{k'-1}) \\ kv_L, & \text{if } t = 1 \end{cases}$$

where $k > k' \in \{1, 2, ...K-1\}$, and t^*_{k-1} satisfies conditions (14a), (14b) and (14c).

The following proposition formalizes our heuristic equilibrium description.

Proposition 3. *Suppose $K \in \mathbb{N}$. There is a $\bar{\Delta} > 0$ such that when $\Delta \in \left(0, \bar{\Delta}\right)$, there is a unique equilibrium in which there is a sequence of fire sale times $\{t^*_k\}_{k=1}^{K-1}$ such that:*

1. *$t^*_{k+1} \leq t^*_k$, and $t^*_k - t^*_{k+1} > \Delta$ when $t^*_k > \Delta$,*
2. *the H-buyers' reservation price is $p_k(t)$ for $t < 1$ and $K(t) = k \in \{1, 2, ...K(0)\}$ and v_H at $t = 1$,*
3. *on the path of play when $K(t) = k$, the seller posts*

$$P(t) = \begin{cases} p_k(t), & \text{if } t < t^*_{k-1}, \\ v_L, & \text{if } t \geq t^*_{k-1} \text{ and } K(t) \geq k. \end{cases}$$

In equilibrium, when $K(t) = k$, the price is $p_k(t)$ for $t < t^*_{k-1}$. Without any transaction, the price smoothly declines and jumps up to $p_{k-1}(t)$ once a transaction happens at t. If there is no transaction before t^*_{k-1}, the price jumps down to v_L, and the price path jumps back to $p_{k-1}(.)$ after a transaction at t^*_{k-1}. Consequently, a highly fluctuating price path can be generated. In Figure 4, we provide some simulation of equilibrium price path.

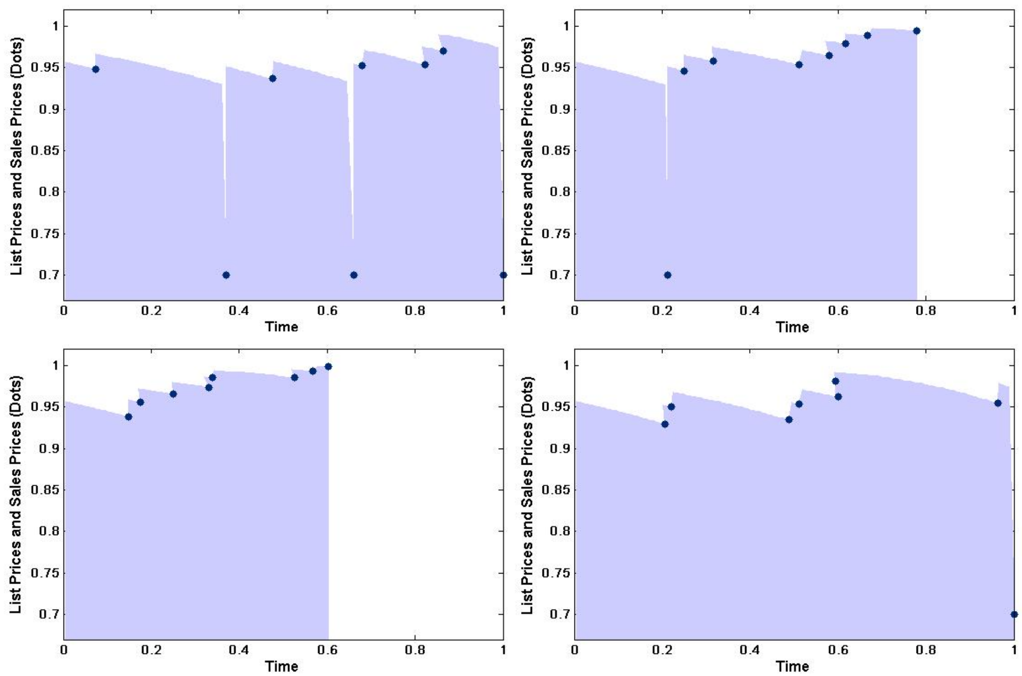

Figure 4: Simulated price path for different realizations of H-buyers' arrival in the 8-unit case. The upper edge of the shaded area describes the equilibrium list price, and dots indicate transactions. The parameter values are $v_H = 1$, $v_L = 0.7$, $M = 10$, $K = 8$ and $\lambda = 7$.

5 Discussion

In this section, we briefly discuss some possible extensions and applications of our baseline model.

5.1 Application: Best Available Rate

In the baseline model, we assume the seller has no commitment power. What if the seller has partial commitment power? In practice, sellers in both the airline and the hotel industries sometimes employ a best available rate (BAR) policy and commit to not posting price lower than this best rate in the future. Does the seller have the incentive to do so in our model? Suppose the seller can commit to not posting a deal before the deadline. Then the seller may benefit. The intuition is as follows. An H-buyer's reservation price depends on the next fire sale time. If there is a deal soon, the reservation price is low, since there is a non-trivial probability that an L-buyer can obtain a good at the fire sale price. At the beginning of the game, if the seller can employ a BAR and commit to not posting v_L before the deadline, he can charge a higher price conditional on the inventory size. To illustrate the idea, we can consider the two-unit case. The seller's payoff by committing $P(t) > v_L$ for $t < 1$ is

$$\Pi_2^{BAR} = \int_0^1 e^{-\lambda s} \lambda \left[p_2(s) + \Pi_1(s) \right] ds + e^{-\lambda} 2 v_L,$$

such that $p_2(t)$ satisfies the ODE (7) with a boundary condition $p_2(1^-) = \frac{M-1}{M+1} v_H + \frac{2}{M+1} v_L$. By committing to no fire sale before the deadline, the seller can ask a higher price when $K(t) = 2$. As a result, $\Pi_2^{BAR} > \Pi_2(0)$ for certain parameters. In Figure 5, we plot the profit with BAR, $\Pi_2^{BAR}(t)$ and that without it, $\Pi_2(t)$. In the beginning $\Pi_2^{BAR}(t) > \Pi_2(t)$. As time goes on, the difference between them vanishes and becomes negative when the time is very close to the deadline.

5.2 Extension: Disappearing H-Buyers

In the baseline model, we assume an H-buyer leaves the market only when her demand is satisfied. Our results do not qualitatively change if buyers leave at a non-trivial rate over time. Suppose a buyer leaves the market at a rate $\rho > 0$ at any time, and her payoff by leaving the market without making a purchase is zero. If a buyer chooses to wait in the market,

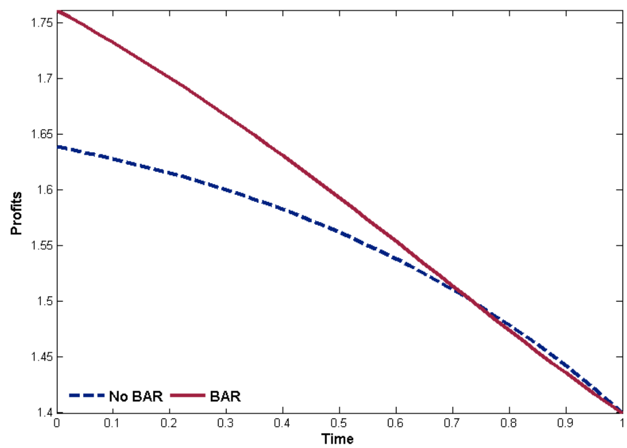

Figure 5: The solid line is the profit with BAR, while the dashed line is that without BAR. When t is close to 0, the profit with BAR is higher than that without BAR. The parameter values are $v_H = 1, v_L = 0.7, M = 3$, and $\lambda = 2$.

she faces the risk of exogenous leaving. In particular, when $K = 1$, an H-buyer's reservation price satisfies the following ODE

$$\dot{p}_1(t) = -(\lambda + \rho)[v_H - p_1(t)] \text{ for } t \in [0, 1),$$

with the boundary condition (2). By rejecting the current offer, an H-buyer needs to take into account two risks: (1) another H-buyer arrives and purchases the first units before her next attention time, and (2) her exogenous departure. Her payoff is zero if either happens.

In the two-unit case, for $t < t_1^*$, the H-buyer's reservation price follows

$$\dot{p}_2(t) = -\lambda[p_1(t) - p_2(t)] - \rho[v_H - p_2(t)],$$

and for $t \geq t_1^*$, the form of $p_2(t)$ is identical to that in the baseline model. The intuition behind it is as follows. For $t < t_1^*$, by rejecting a current offer, an H-buyer needs to take into account the risk that (1) another H-buyer arrives before her next attention time, and (2) she exogenously leaves the market. In the former case, she has to pay $p_1(\tilde{t})$ instead of $p_2(\tilde{t})$ at her next attention time $\tilde{t} > t$; in the latter case, she obtains a payoff of zero, which is equivalent to paying a price v_H. Since the risk of exogenous departure will only change the H-buyer's reservation price qualitatively, our main results still hold.

6 Other Related Literature and Conclusion

6.1 Other Literature

In the *revenue management* literature, in addition to the papers we discuss in section 1.1, there are numerous papers that have examined similar problems in different environments. Gershkov and Moldovanu (2009) extend the benchmark model to the heterogeneous objects case. The standard assumption maintained in these works is that buyers are impatient, and therefore cannot strategically time their purchases. However, as argued by Besanko and Winston (1990), mistakenly treating forward-looking customers as myopic may have an important impact on sellers' revenue. Hence, the revenue management problem with patient buyers draws the economists' attention. For example, Wang (1993) considers the case in which a seller has one object for sale and buyers arrive according to a Poisson distribution and experience a flow delay cost. He shows that with an infinite horizon, the profit-maximizing mechanism is to post a constant price and it may induce a delay of purchases on the path of play.

In a framework similar to that of Board and Skrzypacz (2010), Li (2012) considers a similar model and characterizes the allocation policy that maximizes the expected total surplus and its implementation. Mierendorff (2011a) assumes that buyers randomly arrive and their valuation depends on the time at which the good is sold and characterizes the efficient allocation rule as a generalization of the static Vickrey auction. Pai and Vohra (2010) consider a model without discounting where agents privately arrive and leave the market over time. They show that the revenue-maximizing allocation rule can be characterized as an index rule: each buyer can be assigned an index, and the allocation rule allots the good to a buyer if her index exceeds some threshold. Mierendorff (2011b), on the other hand, considers a similar environment but studies the optimal mechanism design problem when the regularity condition fails. Shneyerov (2012) studies a single-unit revenue management problem where the seller is more patient than the buyers. Su (2007) studies a model where buyers are heterogeneous in both valuation and patience and derives the optimal pricing policy. Deneckere and Peck (2012) study a perfect competitive price posting model where buyers arrive over time. They show that buyers endogenously sort themselves efficiently, with high valuations purchasing first.

In the *durable goods* literature, Stokey (1979, 1981) provides an early discussion of the monopolist's dynamic pricing problem. To consider the issue of new arrivals, Sobel (1991) considers a model with a more general setting and shows that the Coase conjecture does not hold. Sobel (1984) extends the model of Conlisk, Gerstner and Sobel (1984) by considering a multi-seller case. He shows that, in some equilibria, all seller lower their price at the same time and to the same level. Board (2008) allows the entering generations to differ over time. Fuchs and Skrzypacz (2010) study a Coasian bargaining model in which exogenous events (for example, new buyers) may arrive according to a Poisson process. They show that the possibility of arrivals leads to delay. Huang and Li (2012) allow the existence of new arrivals to be initially uncertain but it can be learned by players over time. They show that the interaction between screening and learning about new arrivals can generate frequent price fluctuations when the seller's commitment power vanishes. Mason and Valimaki (2011) study a monopoly pricing problem where a seller faces a sequence of short-lived buyer whose arrival rate is unknown and can be learned over time. Biehl (2001) and Deb (2010) study a durable goods model where consumers' reservation value may change over time. Said (2012) studies a monopoly pricing problem of perishable goods, where buyers arrive over time. He shows that the seller can implement the efficient allocation using a sequence of ascending auctions. McAfee and Wiseman (2008) consider a durable good selling model where the seller can choose the capacity and they show that the Coase conjecture fails. Fuchs and Skrzypacz (2011) study the role of deadlines in a Coasian bargaining model where the seller has a single unit to sell. Dudine, Hendel and Lizzeri (2006) consider a durable good model where demand changes over time and buyers can purchase and store goods in advance. They find that if the seller cannot commit, the prices are higher than in the case in which he can commit, which is inconsistent with the prediction of the standard Coase conjecture literature.

Instead of pricing, many authors study other mechanisms a seller can use to sell her product to strategic buyers. McAfee and Vincent (1997) assume that the seller can run a sequence of auctions and adjust his reservation price over time. Skreta (2006) examines the case where the seller faces one buyer with private valuation in a finite horizon model, allowing the seller to use general mechanisms, and shows that posted prices are revenue-maximizing among all mechanisms. Skreta (2011) extends the model to the case where the seller faces many buyers.

In the *industrial organization* literature, some papers study the role of different kinds of sales. Lazear (1986) studies firms' pricing strategy in a two-period model and provides the first justification of clearance sales. Nocke and Peitz (2007) allow the seller to optimally choose his capacity and price in a two-period model and show that clearance sales may be optimal under certain conditions. Möller and Watanabe (2009) investigate a monopolist's profit-maximizing selling strategy when buyers face uncertainty about their demands. They show that, when aggregate demand exceeds capacity, both advance purchase discounts and clearance sales may be optimal. Lazarev (2012) studies the time paths of prices for airline tickets offered on monopoly routes in the U.S. Using estimates of the model's demand and cost parameters, he compares the welfare consumers receive under the current ticketing system to several alternative systems. In an oligopoly market where sellers face capacity constraint, Kreps and Scheinkman (1983) show that a mixed pricing strategy profile is supported as the equilibrium under certain conditions. Maskin and Tirole (1988) study a duopoly market where firms adjust their price alternately and show that in a Markov perfect equilibrium, the price pattern satisfies the Edgeworth cycle: each firm cuts its price successively to increase its market share until the price war becomes too costly, at which point some firm increases its price. The other firms then follow suit, after which price cutting begins again. In a consumer search model, Varian (1980) justifies the role of sales by a mixed pricing strategy, and Armstrong and Zhou (2011) investigate the role of exploding offers and buy-now discounts.

6.2 Conclusion

This paper makes two contributions. First, we highlight a new channel for generating the periodic fire sales. When the deadline is approaching, the seller, if he still has a large inventory, does not expect many arrivals of high-value buyers, so he has the incentive to liquidate part of his stock via a sequence of fire sales to increase future H-buyers' reservation price. This insight can justify the price fluctuations in industries such as airlines, cruiselines and hotel services. Second, by introducing the inattention frictions of buyers, we provide a tractable framework to study dynamic pricing problems in both finite and infinite horizon games. On the theory side, by introducing the inattention frictions of buyers, one can study a relatively simple equilibrium, the (no-waiting) Markov perfect equilibrium in such games. We believe that the inattention frictions can simplify the analysis in more general environments. On the application side, one can investigate the role of commitment associated with selling strategies, such as the best price guarantee, which is meaningless in a perfect commitment model.

There are many future research projects one can pursue following our work.

Multiple Buyer-Types. In general, considering buyers' multiple reservation values is complicated in our model. Nevertheless, we can discuss a conjecture equilibrium in the three-type case. Specifically, a new buyer arrives with rate λ. Conditional on arrival, the buyer's reservation value of the good is v_H with probability η, and it is v_M with probability $1-\eta$, where $v_H > v_M > v_L$. Similar to the Coase conjecture literature, a skimming property holds; that is, if a price p is acceptable to an M-buyer, it must be acceptable to an H-buyer as well. Define a θ-buyer's reservation price when $K(t) = k$ as $p_k^{\theta}(t)$. The skimming property implies that $p_k^H(t) \geq p_k^M(t)$. At equality, the seller can serve both H-buyers and M-buyers at the same price. Otherwise, the seller can post either $p_k^M(t)$ to serve both, or $p_k^H(t)$ to serve H-buyers only and potentially accumulate M-buyers for a positive measure of time. Over time, if there is no transaction at $p_k^H(t)$, the seller is more and more convinced that there are some M-buyers. If the seller holds a large number of goods and t is close to the deadline, he has the incentive to charge $p_k^M(t)$ to sell a unit to the M-buyer. Similar logic is adopted by Conlisk, Gerstner, and Sobel (1984), albeit in a stationary model. In the case of continuous reservation values, $v \in [v_L, v_H]$, we conjecture that the seller screens buyers smoothly.

Outside Offers and Competition Among Sellers. In the baseline model, we assume that there is a single seller and we extend our results by considering buyers' exogenous departure. However, in the real world, a buyer may leave the market because she finds a better outside offer. Suppose for each buyer, other offers arrive at rate γ, and each offered price $\tilde{p}$ is drawn from a commonly known distribution $F_t(\cdot)$. Hence, an H-buyer's indifference condition at her attention time t implies that

$$\dot{p}_k(t) = -\lambda\left[p_{k-1}(t) - p_k(t)\right] - \gamma F_t(p_k(t))\left\{\mathbb{E}\left[\tilde{p}|\tilde{p} \leq p_k(t)\right] - p_k(t)\right\},$$

where the additional term $\gamma F_t(p_k(t))\left\{\mathbb{E}\left[\tilde{p}|\tilde{p} \leq p_k(t)\right] - p_k(t)\right\}$ reads that: at a rate γ, an outside offer with a price $\tilde{p}$ is realized and, with probability $F_t[p_k(t)]$, it is cheaper than the current price, in which case the buyer takes the offer. Hence, one can easily extend our basic model to consider the effect of outside offers. Furthermore, one can endogenize the distribution by considering a general equilibrium model in which many sellers and buyers randomly match, the arrival and departure rates are interpreted as the search frictions, and the outside offer distribution is given in equilibrium. We conjecture that our mechanism to generate fire sales still holds as long as the competition is not perfect.

Overbooking Policy. In the multi-unit case, we show that the allocation mechanism is generally inefficient. Some L-buyers can obtain goods via fire sales but the H-buyers who arrive late may not. One possible selling strategy to overcome this inefficiency is to allow overbooking. The seller can sell more than he has at a higher price to H-buyers and buy back some goods previously sold to L-buyers. See Courty and Li (2000), Ely, Garrett and Hinnosaar (2012) and Fu, Gautier and Watanabe (2012) for studies of related issues in different environments.

Transparency Policy. In our baseline model, the inventory size is observable. In practice, the inventory size is the seller's private information. However, we can imagine a similar game where the seller can provide verifiable information about his current inventory size without paying any cost. Since the smaller the current inventory size, the higher the price H-buyers are willing to pay, Milgrom's (1981) full disclosure theorem can justify the symmetric information of the inventory size. If the information disclosure is costly, a seller has the incentive to disclose his inventory size only if it is small enough. Another natural question to ask is, if the seller can choose the transparency of his inventory size and past price, is it ex ante optimal to hide this information or not? Is the optimal ex ante transparency policy time-consistent? Recently, Hörner and Vieille (2009), Kim (2012), and Kaya and Liu (2012) study the role of transparency of past prices in different environments and show that it has a significant impact on the formation of future prices.

The Presence of the Secondary Market. In our baseline model, buyers cannot trade with each other. This assumption applies in the airline, cruise and hotel-booking industries, but not in other markets such as sport tickets and theater tickets. We believe that it will be interesting to discuss the role of a secondary market in our framework. See Sweeting (2012) for an empirical analysis of the price dynamics in the secondary markets for major league baseball tickets.

A Appendix: Strategy and Equilibrium

A.1 Admissible Strategy Space

In general, the seller's strategy is a mapping from the set of the seller's history to the price and target sold number.

$$\sigma_S : \mathcal{H}_S \to \mathbb{R}^+ \times \mathbb{N}.$$

For each H-buyer i, let the index function, $a_i(t)$, denote her attention status at t. It is equal to 1 at her attention time, and 0 at other times. At her attention time, the H-buyer can decide to purchase the good or not. At the time she decides to purchase the good, we let the index function, $b_i(t) = 1$; at other times, $b_i(t) = 0$. Let $\omega_i(t) = \{a_i(t), b_i(t)\}$, Ω_i be the set of all $\omega_i(t)$, and $\Omega = \prod_{i=1}^{\infty} \Omega_i$. A non-trivial private history of an H-buyer i at her attention time t is

$$h_{Bi}^t = \left\{ \{a_i(\tau)\}_{\tau=0}^t, \{P(\tau), Q(t), K(\tau)\}_{\tau \in \{\tau' | \tau' \in [0,t], a_i(\tau')=1\}} \right\}$$

and let $\mathcal{H}_B$ represent the set of all non-trivial private histories of an H-buyer. The H-buyers' strategy at their attention time is

$$\sigma_B : \mathcal{H}_B \to [0,1].$$

Denote the underlying outcome by $o(t) = \{P(t), Q(t), K(t), \{\omega_i(t)\}_{i=1}^{\infty}\}$, and let o^t be the underlying history. Given an underlying outcome, players' expected payoff can be calculated.

A metric on the sets of the seller's history is defined as

$$D\left(h_S^t, \tilde{h}_S^t\right) = \int_0^t \left[||P(s), \tilde{P}(s)|| + ||Q(s), \tilde{Q}(s)|| + ||K(s), \tilde{K}(s)|| \right] ds.$$

define the metric on the sets of Ω as follows: for $\omega, \tilde{\omega} \in \Omega$,

$$D_i(\omega_i, \tilde{\omega}_i, [0,t]) = \int_0^t \left[||b_i(s), \tilde{b}_i(s)|| + ||a_i(s), \tilde{a}_i(s)|| \right] ds,$$

and

$$D(\omega, \tilde{\omega}, [0,t]) = \sum_{i=1}^{\infty} D_i(\omega_i, \tilde{\omega}_i, [0,t]).$$

A metric on the sets of the underlying outcome is defined as

$$D\left(o^t, \tilde{o}^t\right) = D\left(h_S^t, \tilde{h}_S^t\right) + D(\omega, \tilde{\omega}, [0,t])$$

where $||\cdot||$ is the Euclidean norm. Let $\mathcal{B}_{\mathcal{H}_S}, \mathcal{B}_{\mathcal{H}_B}$ be Boreal σ-algebra determined by D.

Condition 1. *σ_S is a $\mathcal{B}_{\mathcal{H}_S}$ measurable function and σ_B is a $\mathcal{B}_{\mathcal{H}_B}$ measurable function.*

Condition 2. *For all $t \in [0,1]$ and $h_S^t, \tilde{h}_S^t \in \mathcal{H}_S$ such that $D\left(h_S^t, \tilde{h}_S^t\right) = 0$, $\sigma_S\left(h^t\right) = \sigma_S\left(\tilde{h}^t\right)$.*

The first condition is a technical one, and the second condition implies that, if two seller histories are the same almost surely, the strategy should specify the same price and target supply.

Definition A.1. *A seller's strategy σ_S satisfies the **inertia condition** if given $t \in [0,1)$, there exists an $\varepsilon > 0$ and a constant pricing and supply rule that*

$$\sigma_S\left(\tilde{h}_S^{t'}\right) = \sigma_S\left(h_S^t\right) \quad \forall t' \in (t, t+\varepsilon)$$

for every $\tilde{h}_S^{t+\varepsilon} \in \mathcal{H}$ such that $D\left(h_S^t, \tilde{h}_S^t\right) = 0$.

Note ε is time dependent. This assumption requires that at every time, players must follow a fixed rule specified by σ_S for a small period of time. The key is that the seller's action in $[t, t+\varepsilon)$ cannot depends on each others' action directly almost everywhere. This implies that the seller can vary his actions rule a countably number of times. Let Σ be the set of all σ satisfying inertia conditions.

The following proposition shows that if $\sigma_S \in \Sigma$, a strategy profile (σ_S, σ_B) determines a unique distribution on the underlying outcome space. The spirit of the proof is similar to the proof of Theorem 1 in Bergin and MacLeod (1993).

Proposition A.1. *A strategy profile σ generates a unique distribution over the underlying outcome if $\sigma_S \in \Sigma$.*

Proof. Fixed an underlying outcome o, and $t \in [0,1]$, we want to show that there is a unique distribution $\Gamma \in \Delta\left(\{o_s\}_{s\in[t,1]}\right)$ generated by σ. When $t = 1$, it is trivially given by condition 2. When $t < 1$, the underlying outcome o^t and associated history $h_S^t, \{h_{Bi}^t\}_{i=1}^{\infty}$ are given. Let A_τ be the set of distribution on $\{o_s\}_{s\in[t,\tau)}$ which can be generated by σ. Since players' actions are determined by $\sigma_S\left(h_S^t\right)$ and $\sigma_B\left(h_B^t, P(h_S^t), K(s)\right)$ and the uncertainty is driven by the arrival of new buyers and the selection of the seller when there are sales, A_τ is a singleton when $\tau \in (t, t+\varepsilon)$.

Now we claim A_τ is a singleton for any $\tau \in [t,1]$. Suppose not; then there exists a t^* which is the largest τ such that there is a unique distribution on $\{o_s\}_{s\in[t,\tau)}$ which is generated by σ. By the inertia condition, there is an $\varepsilon > 0$ which depends on τ such that there is a unique distribution on $\{o_s\}_{s\in[\tau,\tau+\varepsilon)}$ generated by σ, which is a contradiction with the definition of t^*.

Since A_τ is nonempty, by the inertia condition, there is a $\varepsilon > 0$ such that $A_{\tau+\varepsilon}$ is a singleton. Proceed iteratively in this way, constructing a unique outcome on $[0,t_1), [0,t_2), ...$ with $t_j > t_j - 1$. The upper bound on this process is at 1; otherwise, there is a contradiction by the definition of the inertia strategy. As a result, there is a unique distribution $\Gamma \in \Delta\left(\{o_s\}_{s\in[t,1]}\right)$ generated by σ.

□

However, the seller cannot respond "instantaneously" to any defection by playing the inertia strategy. So we take the completion of Σ with respect to a metric that measures the underlying outcome distribution induced by a different strategy.

$$d\left(\sigma_S, \sigma'_S\right) = \sup_{t\in[0,1]} \left\{ \sup_{B_t \in \mathcal{F}_t} \left|\Gamma\left(B_t\right) - \Gamma'\left(B_t\right)\right| \right\}$$

where $\{\mathcal{F}_t\}_{t\in[0,1]}$ is the filtration generated by the underlying outcome, so $\mathcal{F}_t$ is the σ-algebra describing the time t outcome, and B_t is a measurable event at time t.

Two inertia strategies are equivalent if they generate the same distribution over the underlying outcome space. Denote by Σ_S^* the completion of Σ_s relative to d. Hence, each strategy $\sigma_S \in \Sigma^*$ corresponds to some Cauchy sequence in Σ. Theorem 2 in Bergin and MacLeod (1993) immediately implies that, for each $\sigma_S \in \Sigma_S^*$, there is a sequence $\{\sigma_S^n\}$ where $\sigma_S^n \in \Sigma_S$ and $\sigma_S^n \to \sigma_S$, such that there is a sequence of distribution $\{\Gamma^n\}$ where $\Gamma^n \in \Delta\left(\{o_\tau\}_{\tau\in[t,1]}\right)$ is the unique distribution generated by σ_S^n, σ_B and $\Gamma^n \to \Gamma \in \Delta\left(\{o_\tau\}_{\tau\in[t,1]}\right)$. Hence, we say σ can be identified with a unique outcome $\Gamma \in \Delta\left(\{o_s\}_{s\in[t,1]}\right)$. We say a strategy σ is *admissible* if and only if $\sigma \in \Sigma^*$. *Q.E.D.*

A.2 On No-Waiting Equilibria

We focus on no-waiting equilibria where buyers believe that no previous H-buyers are waiting in the market both on and off the equilibrium path. We justify this assumption in the following two cases of deviation: "wrong" price and "wrong" inventory size. First, when an H-buyer observes one or more deviation prices, she believes that the seller posts the equilibrium prices always except for at some of her past attention times and the seller's estimation about the population structure of buyers is still the equilibrium one, and therefore the seller would follow the equilibrium pricing rule in the continuation play. The second case is one in which the seller is supposed to post a deal at a sales time. Since there exist many L-buyers who can take the deal immediately, the seller can ensure that his inventory size is consistent with the equilibrium requirement at all times by following the equilibrium strategy. If an H-buyer observes a "wrong" inventory size after the supposed sales time, she knows that there has been a deviation on the seller's side for a positive measure of time, but does not know what prices the seller has been posting. If the prices have been acceptable to H-buyers, there will be no other H-buyers waiting in the market; otherwise, there may be. In fact, we assume that, once an H-buyer observes a "wrong" inventory size, she always believes that the deviation prices have been acceptable to previous H-buyers, and thus, off the path of play, she still believes that no other H-buyers are waiting in the market, and the seller's continuation play is going to be consistent with the equilibrium strategy.

B Appendix: Proofs

B.1 Belief Updating

In this subsection, we derive the law of motion of the seller's belief, $\Phi^+(t)$ and $\Phi^-(t)$.

At $t = l\Delta$, for $n \in \mathbb{N}$, $\Phi_n^-(t) = 0$ for any $l \in \{0, 1, 2, ..1/\Delta\}$. For any $t \in ((l-1)\Delta, l\Delta)$, the updating depends on whether the price at time t is acceptable to the H-buyer who notices it. Given H-buyer's strategy σ_B. As a result, if the price is *not acceptable* to H-buyers in $[t, t+dt)$, Bayes' rule implies that, for any $n \in \mathbb{N}$,

$$
\begin{aligned}
\Phi_n^-(t+dt) \quad = \quad & \Phi_{n-1}^-(t)\left\{\lambda dt + \sum_{n'=1}^{\infty}\Phi_{n'}^+(t)\binom{n'}{1}\left(\frac{dt}{l\Delta - t}\right)^1\left(1-\frac{dt}{l\Delta - t}\right)^{n'-1}\right\} \\
& +\Phi_n^-(t)\left\{1-\lambda dt - \sum_{n'=1}^{\infty}\Phi_{n'}^+(t)\binom{n'}{1}\left(\frac{dt}{l\Delta - t}\right)^1\left(1-\frac{dt}{l\Delta - t}\right)^{n'-1}\right\} \\
& +o(dt),
\end{aligned}
$$

where $\binom{i}{j}\left(\frac{dt}{l\Delta-t}\right)^j\left(1-\frac{dt}{l\Delta-t}\right)^{i-j}$ denotes the probability that j of i H-buyers whose attention times are in $[t, t+dt)$. They notice the offer but decide not to purchase the good. Thus we have the endogenous updating equation of $\Phi^-(t)$:

$$
\dot{\Phi}_n^-(t) = \lim_{dt\to 0}\frac{\Phi_n^-(t+dt) - \Phi_n^-(t)}{dt} = \left[\sum_{n'=1}^{\infty}\frac{n'}{l\Delta - t}\Phi_{n'}^+(t) + \lambda\right]\left[\Phi_{n-1}^-(t) - \Phi_n^-(t)\right].
$$

If the price is *acceptable*, by assuming that buyers follow the equilibrium strategy, there is no change in $\Phi_n^-(t)$.

At $t = l\Delta$, for any $n \in \mathbb{N}$, $\Phi_n^+(t) = \Phi_n^-(t^-)$ for any $l \in \{0, 1, 2, ..1/\Delta\}$. For any $t \in ((l-1)\Delta, l\Delta)$, if there is no transaction, we can derive the law of motion of $\Phi_n^+(t)$ similar to deriving that of $\Phi_n^-(t)$. Also, the law of motion of $\Phi_n^+(t)$ depends on whether the price is acceptable to an H-buyer or not. If the price is *acceptable*, but there is no transaction, then for any $n \in \mathbb{N}$,

$$\begin{aligned}
\dot{\Phi}_n^+(t) &= \lim_{dt\to 0} \frac{\Phi_n^+(t+dt) - \Phi_n^+(t)}{dt} \\
&= \lim_{dt\to 0} \left\{ \frac{\Phi_n^+(t)\left[1-\frac{dt}{l\Delta - t}\right]^n - \Phi_n^+(t)\Sigma_{n'=0}^{\infty}\Phi_{n'}^+(t)\left[1-\frac{dt}{l\Delta-t}\right]^{n'}}{dt\sum_{n'=0}^{\infty}\Phi_{n'}^+(t)\left[1-\frac{dt}{l\Delta-t}\right]^{n'}} \right\} \\
&= \lim_{dt\to 0} \left\{ \frac{-\Phi_n^+(t)}{l\Delta - t} \frac{n - \Sigma_{n'=0}^{\infty}\Phi_{n'}^+(t)\, n' + o(dt)}{\Sigma_{n'=0}^{\infty}\Phi_{n'}^+(t)\left[1-\frac{n'dt}{l\Delta-t}\right] + o(dt)} \right\} \\
&= \frac{-\Phi_n^+(t)}{l\Delta - t}\left[n - \mathbb{E}N^+(t)\right].
\end{aligned}$$

If the price is *not acceptable*, then

$$\begin{aligned}
\dot{\Phi}_n^+(t) &= \lim_{dt\to 0} \frac{\Phi_n^+(t+dt) - \Phi_n^+(t)}{dt} \\
&= \lim_{dt\to 0} \left\{ \frac{\Phi_{n+1}(t)\binom{n+1}{1}\left(\frac{dt}{l\Delta-t}\right)^1\left(1-\frac{dt}{l\Delta-t}\right)^n + \Phi_n(t)\left(1-\frac{dt}{l\Delta-t}\right)^n - \Phi_n^+(t) + o(dt)}{dt} \right\} \\
&= \frac{1}{l\Delta - t}\left[(n+1)\Phi_{n+1}(t) - n\Phi_n(t)\right].
\end{aligned}$$

If there is a transaction at a price $P(t) > v_L$, it must be an H-buyer who makes the purchase, so we have a belief jump following Bayes' rule:

$$\begin{aligned}
\Phi_n^+(t^+) &= \lim_{dt\to 0} \frac{\Phi_{n+1}^+(t)\frac{(n+1)dt}{l\Delta-t} + \Phi_n^+(t)\lambda dt + o(dt)}{\sum_{n'=0}^{\infty}\Phi_{n'}^+(t)\left(\frac{n'dt}{l\Delta-t} + \lambda dt\right) + o(dt)} \\
&= \frac{(n+1)\Phi_{n+1}^+(t) + \Phi_n^+(t)\lambda(l\Delta - t)}{\sum_{n'=0}^{\infty}\left[\Phi_{n'}^+(t)\,n'\right] + \lambda(l\Delta - t)} \\
&= \frac{(n+1)\Phi_{n+1}^+(t) + \Phi_n^+(t)\lambda(l\Delta - t)}{\mathbb{E}\left[N^+(t)\right] + \lambda(l\Delta - t)},
\end{aligned}$$

for any $n \in \mathbb{N}$.

If there is a transaction at a deal price $P(t) \leq v_L$, it may be an L-buyer or an H-buyer who made the purchase, so the updating of $\Phi(t)$ would depend on the current belief of the number of L-buyers. Let $\Upsilon_m(t)$ denote the seller's belief that $M(t) = m$ at time t. At the beginning of each period, $\Upsilon_M(t) = 1$, but $M(t)$ may change within a period because some L-buyers may leave the market by making their purchases at deal prices. Within a period, after the first deal at time t, we have

$$\Upsilon_M\left(t^+\right) = \sum_{n=0}^{\infty} \Phi_n(t) \frac{n}{M+n}, \Upsilon_{M-1}\left(t^+\right) = \sum_{n=0}^{\infty} \Phi_n(t) \frac{M}{M+n}, \tag{B.1}$$

and $\Upsilon_{M-i}(t^+) = 0$ for $i = 2, 3, ...M$. After the k^{th} deal at time t, we have:

$$\Upsilon_M\left(t^+\right) = \Upsilon_M(t) \sum_{n=0}^{\infty} \Phi_n(t) \frac{n}{M+n}, \tag{B.2}$$

$$\Upsilon_{M-i}\left(t^+\right) = \Upsilon_{M-i}(t) \sum_{n=0}^{\infty} \Phi_n(t) \frac{n}{M-i+n} + \Upsilon_{M-i+1}(t) \sum_{n=0}^{\infty} \Phi_n(t) \frac{M-i+1}{M-i+1+n}, \tag{B.3}$$

for $i = 1, 2, ...k$, and

$$\Upsilon_{M-k-i}\left(t^+\right) = 0 \text{ for } i = 1, 2, ..M-k. \tag{B.4}$$

Similarly, the belief of $N^-(t)$ and $N^+(t)$ will also jump as follows:

$$\begin{aligned}
\Phi_n^+\left(t^+\right) &= \Phi_n^+(t) \sum_{n'=0}^{\infty} \sum_{m=0}^{M} \Phi_{n'}^-(t) \Upsilon_m(t) \frac{m+n'}{n+m+n'} \\
&\quad + \Phi_{n+1}^+(t) \sum_{n'=0}^{\infty} \sum_{m=0}^{M} \Phi_{n'}^-(t) \Upsilon_m(t) \frac{n+1}{n+1+m+n'}, \\
\Phi_n^-\left(t^+\right) &= \Phi_n^-(t) \sum_{n'=0}^{\infty} \sum_{m=0}^{M} \Phi_{n'}^+(t) \Upsilon_m(t) \frac{m+n'}{n+m+n'} \\
&\quad + \Phi_{n+1}^-(t) \sum_{n'=0}^{\infty} \sum_{m=0}^{M} \Phi_{n'}^+(t) \Upsilon_m(t) \frac{n+1}{n+1+m+n'},
\end{aligned}$$

for any $n \in \mathbb{N}$.

The law of motion of the seller's belief can be summarized in the following proposition. Denote t_k^d be the d^{th} deal times within the period.

Proposition B.1. *Let $P(t)$ be the price at t. The seller's beliefs $\Phi^+(t)$ and $\Phi^-(t)$ update as follows: for any $n \in \mathbb{N}$,*

1. *at $t = l\Delta$, $\Phi_n^-(t) = 0$. For any $t \in ((l-1)\Delta, l\Delta)$, $\Phi^-(t)$ smoothly evolves s.t.*

$$\dot{\Phi}_n^-(t) = [1 - \sigma_B(P(t))]\left[\sum_{n'=1}^{\infty} \frac{n'}{l\Delta - t}\Phi_{n'}^+(t) + \lambda\right]\left[\Phi_{n-1}^-(t) - \Phi_n^-(t)\right],$$

2. *at $t = l\Delta$, $\Phi_n^+(t) = \Phi_n^-(t^-)$. For any $t \in ((l-1)\Delta, l\Delta)$, if there is no transaction, $\Phi^+(t)$ smoothly evolves s.t.*

$$\begin{aligned}\dot{\Phi}_n^+(t) &= [1 - \sigma_B(P(t))]\left\{\frac{1}{l\Delta - t}[(n+1)\Phi_{n+1}(t) - n\Phi_n(t)]\right\} \\ &\quad - \sigma_B(P(t))\frac{\Phi_n^+(t)}{l\Delta - t}\left[n - \mathbb{E}N^-(t)\right],\end{aligned}$$

3. *at $t \in [0,1)$, if there is a transaction at a price $P(t) \leq v_L$, $\Phi^+(t)$ and $\Phi^-(t)$ jump as follow:*

$$\begin{aligned}\Phi_n^+\left(t^+\right) &= \Phi_n^+(t)\sum_{n'=0}^{\infty}\sum_{m=0}^{M}\Phi_{n'}^-(t)\Upsilon_m(t)\frac{m+n'}{n+m+n'} \\ &\quad + \Phi_{n+1}^+(t)\sum_{n'=0}^{\infty}\sum_{m=0}^{M}\Phi_{n'}^-(t)\Upsilon_m(t)\frac{n+1}{n+1+m+n'}, \\ \Phi_n^-\left(t^+\right) &= \Phi_n^-(t)\sum_{n'=0}^{\infty}\sum_{m=0}^{M}\Phi_{n'}^+(t)\Upsilon_m(t)\frac{m+n'}{n+m+n'} \\ &\quad + \Phi_{n+1}^-(t)\sum_{n'=0}^{\infty}\sum_{m=0}^{M}\Phi_{n'}^+(t)\Upsilon_m(t)\frac{n+1}{n+1+m+n'}\end{aligned}$$

   where $\Upsilon_m(t)$ is the seller's belief about $M(t)$ and its law of motion is given in (B.1), (B.2), (B.3),and (B.4) for the k^{th} deal, and

4. *for any $l = 1, 2, ..1/\Delta$, $t \in [(l-1)\Delta, l\Delta)$, if there is a transaction at a price $P(t) > v_L$, $\Phi^+(t)$ jumps as follows:*

$$\Phi_n^+\left(t^+\right) = \frac{(n+1)\Phi_{n+1}^+(t) + \Phi_n^+(t)\lambda(l\Delta - t)}{\mathbb{E}[N^+(t)] + \lambda(l\Delta - t)},$$

   where $\Phi_{-1}^-(t) = 0$, $l \in \{0, 1, 2, ..1/\Delta\}$.

B.2 Proofs for the Single-Unit Case

B.2.1 Equilibria Construction

We construct an equilibrium such that the following conditions hold: (1) the seller posts a price $P(t)$ such that an H-buyer is indifferent between taking and leaving it for $t < 1$, (2) an H-buyer makes the purchase once she arrives, and (3) $P(1) = v_L$ is posted at the deadline. Construct the H-buyers' reservation price. At the deadline, it is obviously v_H. Since the seller posts $P(1) = v_L$ in any equilibrium, at $t \in [1-\Delta, 1)$, the H-buyers' reservation price is

$$\begin{aligned} v_H - p_1(t) &= e^{-\lambda(1-t)} \frac{v_H - v_L}{M+1}, \\ \dot{p}_1(t) &= -\lambda [v_H - p_1(t)]. \end{aligned}$$

As $t \to 1$, $p_1(t) \to p_1(1^-)$. Differentiating $p_1(t)$ yields

$$\dot{p}_1(t) = -\lambda e^{-\lambda(1-t)} \frac{v_H - v_L}{M+1} = -\lambda [v_H - p_1(t)]$$

with a boundary condition $p_1(1^-)$ at $t = 1$. Let $U_{1-\Delta}$ be an H-buyer's expected payoff at the beginning of the last period. The expectation is over the random attention time, and the risk of arrival of new buyers. Hence

$$\begin{aligned} U_{1-\Delta} &= \int_{1-\Delta}^{1} \frac{1}{\Delta} e^{-\lambda(s-1+\Delta)} [v_H - p_1(s)] \, ds \\ &= e^{-\lambda\Delta} \frac{v_H - v_L}{M+1} = v_H - p_1(1-\Delta). \end{aligned}$$

Consider a t that is smaller than but arbitrarily close to $1-\Delta$. At this attention time, an H-buyer's reservation price is

$$\begin{aligned} v_H - p_1(t) &= e^{-\lambda(1-\Delta-t)} U_{1-\Delta}, \\ \dot{p}_1(t) &= -\lambda [v_H - p_1(t)]. \end{aligned}$$

As $t \to 1-\Delta$, we have $\lim_{t \nearrow 1-\Delta} p_1(t) = p_1(1-\Delta)$, thus $p_1(t)$ is differentiable at $1-\Delta$. Repeating the above argument for $1/\Delta$ times, the reservation price $p_1(t)$ is differentiable in $[0, 1)$ and satisfies the ODE (5) with the boundary condition (2).

The deal price is posted at the deadline only, and H-buyers do not delay their purchases, so neither the H-buyers' reservation price nor the seller's equilibrium profit depends on Δ. The closed-form solution of $p_1(t)$ and $\Pi_1(t)$ are given by

$$\begin{aligned} p_1(t) &= v_H - \frac{v_H - v_L}{M+1} e^{-\lambda(1-t)}, \\ \Pi_1(t) &= \left[1 - e^{-\lambda(1-t)}\right] v_H + e^{-\lambda(1-t)} v_L - \frac{v_H - v_L}{M+1} e^{-\lambda(1-t)} \lambda (1-t). \end{aligned}$$

In sum, the equilibrium strategy profile (σ_S^*, σ_B^*) is given as follows. The seller's equilibrium strategy $\sigma_S^*(t, \Phi^-(t), \Phi^+(t)) = p_1(t)$ for any $[\Phi^-(t), \Phi^+(t)] \in \Xi_S$ and $t < 1$, and $\sigma_S^*(1) = v_L$. The H-buyers' equilibrium strategy σ_B^* satisfies $\sigma_B^* = \mathbf{1}_{\{P(t) \leq p_1(t), t \in [0,1)\}} + \mathbf{1}_{[P(1) \leq v_H]}$.

B.2.2 The Proof of Proposition 1

We prove Proposition 1 step by step. A simple observation is that, given the seller's equilibrium strategy, H-buyers do not have an incentive to deviate since they are indifferent everywhere. To ensure the existence of the conjecture equilibrium, we only need to rule out deviations by the seller. We show that the seller has no incentive to post unacceptable prices for a positive measure of time. As a result, the seller has no profitable deviation. Since the construction of $p_1(t)$ is unique, there is no other equilibria in addition to the equilibrium we proposed.

Suppose the seller follows the equilibrium strategy. His expected profit satisfies the following equation:

$$\Pi_1(t) = \int_t^1 e^{-\lambda(s-t)} \lambda p_1(s)\, ds + e^{-\lambda(1-t)} v_L.$$

Taking the derivative with respect to time yields,

$$\dot{\Pi}_1(t) = -\lambda e^{-\lambda(1-t)} (v_H - v_L) \frac{M + \lambda(1-t)}{M+1} < 0.$$

Now we show that the seller's best response is indeed to post $P(t) = p_1(t)$ for $t < 1$ and $P(1) = v_L$. The proof is given step by step.

Step 1. At the deadline, it is the seller's dominant strategy to post $P(1) = v_L$. **Step 2**. At any time, a price $P(t) < p_1(t)$ is dominated by $p_1(t)$. **Step 3**. We claim that the seller has no incentive to post unacceptable prices for a positive measure of time. Suppose not, and the seller posts $P(t) > p_1(t)$ for $t \in [t', t'')$. We claim that such strategy is dominated by an alternative strategy: replacing $P(t)$ by $p_1(t)$ for $t \in [t', t'')$ but keep playing the original continuation strategy. To see the reason, consider two cases. **Case 1**: there is no arrival in $[t, t')$. In this case, the seller is indifferent between two strategies. **Case 2**: some H-buyers arrive in $[t, t')$. In this case, if the seller adopts the original strategy, his expected payoff is less than $p_1(t'')$, since (1) the H-buyer's reserve (acceptable) price before the deadline, $p_1(\cdot)$, is decreasing in time and (2) the seller's payoff at the deadline is v_L. If the seller adopts the alternative strategy, his expected payoff is $p_1(\tau_1)$ where τ_1 is a random time at which the first H-buyer arrives in $[t, t')$. Since $p_1(\tau_1) \geq p_1(t'')$, for any history, the original strategy is dominated by the alternative one. In general, the argument ensures that the seller has no incentive to post unacceptable prices in finite many positive measure time-intervals. Hence, the seller has no incentive to adopt the deviation strategy by posting unacceptable price for a positive measure of time. Apparently, any $P(t) < p_1(t)$ for a positive measure of time is dominated by the equilibrium pricing rule. Consequently, it is the seller's best response to post $P^*(t) = p_1(t)$ for $t < 1$, and $P^*(t) = v_L$, and our conjecture equilibrium is an equilibrium. By construction, $p_1(t)$ is unique, so there is no other equilibrium. *Q.E.D.*

B.3 Proofs for the Two-Unit Case

At any t such that $K(t) = 1$, the problems are the same as in the case where $K = 1$; hence, $p_1(t)$ and $\Pi_1(t)$ remain in the same form, and so does $U^1_{l\Delta}$. At $t = 1$ and $K(1) = 2$, the seller posts $p_2(1) = v_L$ for sure. Now we need to look at the case where $t < 1$ and $K(t) = 2$.

B.3.1 The Proof of Lemma 1

Suppose not. Since v_L is posted only at the deadline, the seller's equilibrium profits at the deadline are given by

$$\Pi_k(1) = kv_L, k = 1, 2.$$

and $p_k(t)$, the reservation price at $k = 1, 2$, is post to serve H-buyers only at any $t < 1$. Specifically,

$$\begin{aligned} p_2(t) &= v_H - \frac{v_H - v_L}{M+1} e^{-\lambda(1-t)} \left[2 + \lambda(1-t)\right], \text{ and} \\ p_1(t) &= v_H - \frac{v_H - v_L}{M+1} e^{-\lambda(1-t)} \end{aligned}$$

Define $\tilde{\Pi}_2(t)$ as the seller's profit if $p_2(t)$ is always posted when $t<1$ and $K(t)=2$, then

$$\begin{aligned}\tilde{\Pi}_2(t) &= \int_t^1 \lambda e^{-\lambda(s-t)}\left[p_2(s)+\Pi_1(s)\right]ds + 2v_L e^{-\lambda(1-t)} \\ &= 2v_H - 2(v_H-v_L)e^{-\lambda(1-t)} \\ &\quad -\frac{v_H-v_L}{M+1}e^{-\lambda(1-t)}\left[\lambda(1-t)(M+3)+\lambda^2(1-t)^2\right].\end{aligned}$$

Immediately,

$$\begin{aligned}&\tilde{\Pi}_2(t) - \left[v_L+\Pi_1(t)\right] \\ =\ & (v_H-v_L)\left(1-2e^{-\lambda(1-t)}\right) \\ &+\frac{v_H-v_L}{M+1}e^{-\lambda(1-t)}\left[M+1-\lambda(1-t)(M+2)-\lambda^2(1-t)^2\right].\end{aligned}$$

Though this difference is not monotone, using a Taylor expansion and algebra, there are two cases: (i) either $\tilde{\Pi}_2(t)-\left[v_L+\Pi_1(t)\right]<0$ for all $t<1$ when $\tilde{\Pi}_2(0)<v_L+\Pi_1(0)$, (ii) or, if $\tilde{\Pi}_2(0)>v_L+\Pi_1(0)$, $\exists\ t^*<1$ s.t. $\tilde{\Pi}_2(t^*)=v_L+\Pi_1(t^*)$ and $\tilde{\Pi}_2(t)<v_L+\Pi_1(t)$ for $t\in(t^*,1)$. *Q.E.D.*

B.3.2 The Proof of Proposition 2

Equilibrium Construction. We first construct the H-buyers' reservation price. Suppose that all buyers believe that the seller posts a deal $p_2(t_1^*)=v_L$ at $t_1^*<1$ if $K(t_1^*)=2$, the H-buyer's reservation price $p_2(t)$ before t_1^* if $K(t)=2$, and $p_1(t)$ at any t s.t. $K(t)=1$. Any H-buyer believes that she is the only one in the market and she accepts any price that is not higher than the reservation price. Similar to the single unit case, the H-buyer's reservation price $p_2(t)$ when $t\in[t_1^*,1)$ satisfies:

$$v_H - p_2(t) = \frac{1}{M+1}(v_H-v_L)+\frac{M}{M+1}e^{-\lambda(l\Delta-t)}U_{l\Delta}^1,$$

where, as in the single-unit case, $U_{l\Delta}^1 = e^{-\lambda(1-l\Delta)}\frac{v_H-v_L}{M+1}$ and $t\in[(l-1)\Delta, l\Delta)$ for some $l<1/\Delta$, hence

$$\begin{aligned}p_2(t) &= \frac{Mv_H+v_L}{M+1}-\frac{M}{(M+1)^2}(v_H-v_L)e^{-\lambda(1-t)} \\ &= \frac{M}{M+1}p_1(t)+\frac{1}{M+1}v_L,\ \text{ for } t\in[t_1^*,1-\Delta).\end{aligned}$$

Observe that $\dot{p}_2 = M/(M+1)\dot{p}_1$ for $t \in [t_1^*, 1-\Delta)$. For $t \in [1-\Delta, 1)$,

$$v_H - p_2(t) = \frac{1}{M+1}(v_H - v_L) + \frac{M}{M+1} e^{-\lambda(1-t)} \frac{v_H - v_L}{M},$$

and $p_2(t) < \frac{M}{M+1} p_1(t) + \frac{1}{M+1} v_L$ since no new L-buyer will enter and an H-buyer's reservation price is $\tilde{p}_1(t) = v_H - e^{-\lambda(1-t)} \frac{v_H - v_L}{M}$ in this case. To construct the equilibrium, we study the auxiliary problem in which $p_2(t) = \frac{M}{M+1} p_1(t) + \frac{1}{M+1} v_L$ for $t \in [t_1^*, 1)$, and show that the seller's optimal fire sale time is $t_1^* < 1-\Delta$ in this auxiliary problem when Δ is small. Furthermore, we argue that the seller's optimal fire sale time is also equal to t_1^* in the problem where $p_2(t) = \frac{1}{M+1}(v_H - v_L) + \frac{M}{M+1} e^{-\lambda(1-t)} \frac{v_H - v_L}{M}$ for $t \in [1-\Delta, 1)$.

If $t < t_1^*$, then for some l, $t \in [(l-1)\Delta, l\Delta) \cap [0, t_1^*)$. If $l\Delta \geq t_1^*$, then $p_2(t)$ satisfies:

$$v_H - p_2(t) = e^{-\lambda\left(t_1^* - t\right)} U_{t_1^*}^2 + \lambda(t_1^* - t) e^{-\lambda\left(t_1^* - t\right)} e^{-\lambda\left(l\Delta - t_1^*\right)} U_{l\Delta}^1,$$

where $U_{t_1^*}^2 = v_H - p_2(t_1^*)$. Otherwise, if $l\Delta < t_1^*$, then

$$v_H - p_2(t) = e^{-\lambda(l\Delta - t)} U_{l\Delta}^2 + \lambda(l\Delta - t) e^{-\lambda(l\Delta - t)} U_{l\Delta}^1,$$

where $U_{l\Delta}^2 = v_H - p_2(l\Delta)$. In either of the two cases, we have

$$p_2(t) = v_H - \frac{v_H - v_L}{M+1} e^{-\lambda(1-t)} \left[e^{\lambda\left(1 - t_1^*\right)} + \frac{M}{M+1} + \lambda(t_1^* - t) \right] < p_1(t),$$

for $t \in [0, t_1^*)$, and $\dot{p}_2(t) = -\lambda(p_1(t) - p_2(t))$ for $t \in [0, t_1^*)$. Note that $p_2(\cdot)$ is continuous on $[0, 1]$.

In fact, for any buyer's belief on t_1^*, $p_2(\cdot)$ depends on t_1^* through the boundary condition at t_1^* only but does not depend on Δ.

Now we consider the seller's problem. Given the buyer's reservation price $p_2(\cdot)$ based on the belief of t_1^*, the seller chooses the actual deal time, with $p_2(\cdot)$ forced to be the pricing strategy before the deal time. Hence,

$$\Pi_2(t) = \max_{t_1} \int_t^{t_1} e^{-\lambda(s-t)} \lambda [p_2(s) + \Pi_1(s)] ds + e^{-\lambda(t_1 - t)} [v_L + \Pi_1(t_1)]. \tag{B.5}$$

In equilibrium, the buyers' belief is correct, so the seller's optimal choice is indeed t_1^*. The first derivative w.r.t. t_1 at $t_1^*$is

$$\begin{aligned} & e^{-\lambda\left(t_1^* - t\right)} \lambda [p_2(t_1^*) - v_L] + e^{-\lambda\left(t_1^* - t\right)} \dot{\Pi}_1(t_1^*) \\ = \; & \lambda e^{-\lambda\left(t_1^* - t\right)} [p_2(t_1^*) - v_L - p_1(t_1^*) + \Pi_1(t_1^*)] = 0 \end{aligned}$$

Or equivalently, $p_2(t_1^*) - v_L - p_1(t_1^*) + \Pi_1(t_1^*) = 0$.

Define $f(\cdot)$ on $[0,1]$ as follows:

$$f(t) = p_2(t) - v_L - p_1(t) + \Pi_1(t).$$

For $t \geq t_1^*$, we have $p_2(t_1) - p_1(t_1) = \frac{1}{M+1}[v_L - p_1(t_1)]$, then

$$\begin{aligned} f(t) &= \Pi_1(t) - v_L - \frac{p_1(t) - v_L}{M+1} \\ &= \frac{v_H - v_L}{M+1}\left\{M - e^{-\lambda(1-t)}\left[M + \frac{M}{M+1} + \lambda(1-t)\right]\right\} \end{aligned}$$

Obviously, $\dot{f}(t) < 0$ and $f(1) = -M/(M+1) < 0$. Define t_1^* as the unique solution to $f(t) = 0$ if it exists, otherwise define $t_1^* = 0$. By construction, for $t \in (t_1^*, 1)$, the optimal solution of (B.5) is t; thus, the seller does not have any incentive to choose a deal time later than t_1^* in the auxiliary problem. If $t_1^* > 0$ i.e. $f(t_1^*) = 0$, for $t < t_1^*$, $\dot{p}_2(t) = -\lambda(p_1(t) - p_2(t))$, hence

$$f(t) = \frac{1}{\lambda}\dot{p}_2(t) + \Pi_1(t) - v_L,$$

and $\dot{f}(t) = \dot{p}_2(t) + \dot{\Pi}_1(t) - \dot{p}_1(t)$ in which $\dot{p}_2(t) < 0$ and $\dot{\Pi}_1(t) - \dot{p}_1(t) = \lambda e^{-\lambda(1-t)}\frac{v_H - v_L}{M+1}[1 - \lambda(1-t) - M] < 0$, therefore $\dot{f}(t) < 0$ for $t < t_1^*$. Since p_2, p_1 and Π_1 are all continuous over $[0,1]$, we have a continuous $f(t)$ and $\lim_{t \nearrow t_1^*} f(t) = f(t_1^*) = 0$, consequently $f(t) > 0$ for $t < t_1^*$; thus, the seller does not have any incentive to choose a deal time earlier than t_1^*.

Suppose Δ is small; thus, after the fire sale at t_1^*, new L-buyers enter and their number is M at the deadline. Hence, after the fire sale, the H-buyer's reservation price for $t \in (t_1^*, 1)$ is $p_1(t)$. Off the path of play, the story is different. Case 1. Suppose the seller holds the fire sales at $t < 1 - \Delta$. Then new L-buyers enter before the deadline, and the H-buyer's reservation price is still $p_1(t)$. Case 2. Suppose the seller runs the fire sale at $t \in (1 - \Delta, 1)$. Then there is no new L-buyer enters after the sales. Hence, the H-buyer's reservation price is $\tilde{p}_1(t) = v_H - e^{-\lambda(1-t)}\frac{v_H - v_L}{M}$ after the fire sale, and $\tilde{p}_2(t) = v_H - \frac{1}{M+1}(v_H - v_L) - \frac{M}{M+1}e^{-\lambda(1-t)}\frac{v_H - v_L}{M}$ before the fire sale. Since $\tilde{p}_k(t) < p_k(t)$, the seller's profit by running the fire sale after $1 - \Delta$ is strictly less than that in the auxiliary problem. Hence, it is strictly dominated. Consequently, in the real problem, the seller does not have any incentive to choose a deal time later than t_1^*.

Verification of the Conjecture. Next, we need to verify, when the seller can freely choose any price at any time, whether our conjecture equilibrium is indeed an equilibrium. By construction, H-buyers have no incentive to deviate. First, we show that the seller has no incentive to deviate from $p_2(t)$ when $K(t) = 2$. We then show that there is no other equilibrium in addition to equilibrium we proposed.

First, by the proof of proposition 1, when $K(t)$ jumps to 1, the continuation play of the seller in any equilibrium is $P(t) = p_1(t)$ for $t < 1$ and $P(1) = v_L$. Second, a simple observation is that, when $K(t) = 2$, any strategy induces $P(t) \in (v_L, p_2(t))$ is dominated by $p_2(t)$, so it is suboptimal. Third, conditional on k, the H-buyer's reservation price declines over time. Different from the single-unit case, the seller can enhance the H-buyer's reservation price in future by reducing his inventory. As a result, the seller may have the incentive to accumulate H-buyers by charging $P(t) > p_2(t)$ when $K(t) = 2$, and charge them $p_1(\cdot)$ after a fire sale. However, we claim that, when $K(t) = 2$, the seller has no incentive to choose a strategy with a price $P(t) > p_2(t)$ when $K(t) = 2$ at any positive measure of time.

Given any seller's strategy, denote $t_1^d = \inf\{t|K(t) = 2, P(t) = v_L\}$. Consider the last period first. For $t \in [1-\Delta, 1]$, it is obvious that the seller's optimal price is either $p_2(t)$ or v_L when $K(t) = 2$. The reason is that it is the last chance that H-buyers will accept a price greater than v_L, and there is no benefit to posting unacceptable prices. Next we claim that, given H-buyers' reservation price, the seller's best response satisfies the following properties: for $t < t_1^d$, $P(t) = p_2(t)$ when $K(t) = 2$. We verify this step by step.

Step 1. Suppose in the seller's best response, $t_1^d \in [(l-1)\Delta, l\Delta)$. We call this period the fire sale period. We claim that for $t \in [(l-1)\Delta, t_1^d)$, $P(t) = p_2(t)$ when $K(t) = 2$. Suppose not. Then there are countably many time intervals with a positive measure in the current period in which the seller posts $P(t) > p_2(t)$ when $K(t) = 2$. We call them non-selling time intervals. **Case 1**. $P(t) > p_2(t)$ for $t \in [(l-1)\Delta, t_1^d)$. By doing so, the benefit is to accumulate H-buyers whose attention times are in such intervals and induce them to accept high prices after the fire sale. However, such a pricing strategy is dominated by the following one: posting v_L at $(l-1)\Delta$ and $p_1(t)$ for $t > (l-1)\Delta$. The reasons are that (1) $p_1(t)$ is decreasing over time, (2) an H-buyer who arrives at $t \in ((l-1)\Delta, t_1^d)$ is the only H-buyer in the market, and he may take the deal at t_1^d instead of paying $p_1(\tilde{t})$ at her next attention time with positive probability. Hence, we have a contradiction! **Case 2**. Suppose there is a $t' \in [(l-1)\Delta, t_1^d)$ such that $P(t) = p_2(t)$ for $t \in [(l-1)\Delta, t')$ and $P(t) > p_2(t)$ for $t \in [t', t_1^d)$ when $K(t) = 2$. Similar to the argument in case 1, the seller can post v_L at t' instead of at t_1^d and earn extra benefit. **Case 3**. There are countably many mutually exclusive subintervals of $[(l-1)\Delta, t_1^d)$ in which $P(t) > p_2(t)$. Then there is a $t'' \in [(l-1)\Delta, t_1^d)$ such that $t_1^d - t''$ equals the measure of the sum of those in the non-selling time intervals. Each H-buyer's attention time follows an independent uniform distribution, and newly arrived H-buyers' arrival rate is time-independent, so the population structure of H-buyers whose attention times are in $[t'', t_1^d)$ is identical to that in the non-selling time intervals. Since both $p_2(t)$ and $p_1(t)$ decrease over time, the original pricing strategy is dominated by the following one at $t = (l-1)\Delta$: the seller posts $p_2(t)$ for $t \in [(l-1)\Delta, t')$ and $P(t) > p_2(t)$ for $t \in [t', t_1^d)$. Then, by the logic of case 2, we have a contradiction! In short, the seller does not post $P(t) > p_2(t)$ for $t \in [(l-1)\Delta, t_1^d)$.

Step 2. Now we claim that for any $t \in [(l-2)\Delta, (l-1)\Delta)$, the seller's best response satisfies that $P(t) = p_2(t)$. Suppose not. By the same argument in case 3 of step 1, we can focus on the strategy where $P(t) = p_2(t)$ for $t \in [(l-2)\Delta, t'')$ and $P(t) > p_2(t)$ for $t \in (t'', (l-1)\Delta)$. Then there must exist a $t' \in [(l-1)\Delta, t'')$ such that at time t the expected distribution of the number of H-buyers whose attention times are in $[t', \frac{t'+t_d^1}{2})$ equals that in $[\frac{t'+t_d^1}{2}, t_1^d)$. Two intervals have the same length, so the process of the attention times is identical too. As a result, we claim that the original strategy is dominated by the following one at time t: the seller posts $p_2(t)$ for $t \in [t', \frac{t'+t_d^1}{2})$ but $P(t) > p_2(t)$ for $t \in [\frac{t'+t_d^1}{2}, t_1^d)$. Again, by the logic of case 2 in step 1, we have a contradiction!

Step 3. In the period before the fire sales period, the price is acceptable and each H-buyer observes the price in that period. Thus, for periods before the seller has no incentive to post $P(t) > p_2(t)$. Hence, in the seller's best response, $P(t) = p_2(t)$ or v_L when $K(t) = 2$. By the construction of the auxiliary problem, we know the optimal fire sale time is t_1^*, thus the seller has no incentive to deviate from his equilibrium strategy.

Uniqueness. Since $t_1^*$is uniquely constructed, there is no other equilibrium. *Q.E.D.*

B.4 Proof of Proposition 3

B.4.1 Equilibria Construction

We construct the equilibrium by induction. Suppose there is a unique equilibrium for the game where $K(0) = K$ in which there exists a sequence of $\{t_k^*\}_{k=1}^{K-1}$, and $p_k(t)$ for $k = \{1, 2, ...K\}$, such that $t_{k+1}^* < t_k^*$, $p_{k+1} < p_k$, and $\dot{p}_k < 0$ where differentiable. Consequently, by the indifference conditions of an H-buyer's reservation price and uniform distributed attention time in a period, we can define $U_{l\Delta}^k = v_H - p_k(l\Delta)$ as the expected utility of an

H-buyer if her next attention time is in next period starting from $l\Delta < t^*_{k-1}$ and $K(l\Delta) = k$, and $U^k_{t^*_{k-1}} = v_H - p_k\left(t^*_{k-1}\right)$ the expected utility if the next attention time is t^*_{k-1} and $K\left(t^*_{k-1}\right) = k$. We construct the candidate equilibrium for the game where $K(0) = K+1$, which includes: the H-buyers reservation price $p_{K+1}(t)$, the equilibrium first fire sale time t^*_K, and the seller' pricing strategy.

When $t^*_{K-1} = 0$, $t^*_K = 0$ as well. When $t^*_{K-1} < 0$, similar to the two-unit case, we can construct a fire sale time $t^*_K \in [0, t^*_{K-1}]$. Suppose buyers believe that the seller posts deals at $0 \le t^*_K \le t^*_{K-1} < ... < t^*_1 < 1$ when $K(t^*_k) > k$ and posts the H-buyer's reservation price $p_k(t)$ when $K(t) = k$, $k = 1, ..., K+1$. We consider the case where Δ is small enough. We assume $\forall k$, $\exists l_k$ s.t. $t^*_k < l_k\Delta < t^*_{k-1}$, that is, there is at most one deal time in a period. We will verify this hypothesis later.

First, consider $t \ge t^*_K$. Trivially, the seller will post $p_K(1) = v_L$ at the deadline and the reservation price of an H-buyer is v_H. When $t \in [t^*_{K-i}, t^*_{K-i-1})$, and $K(t) = K+1$, an H-buyer expects the seller to post v_L immediately and to reduce his inventory to $K-i$, hence

$$p_{K+1}(t) = \frac{i+1}{M+1}v_L + \frac{M-i}{M+1}p_{k-i}(t)$$

for $i = 0, ..., K-1$ and $t^*_0 := 1$. Note that, when $t > t^*_K$, $p_{K+1}(t)$ is decreasing but not continuous because $\lim_{t \nearrow t^*_k} p_{K+1}(t) > p_{K+1}(t^*_k)$, $\forall k < K$ and $\dot{p}_{K+1} = (M-i)/(M+1)\,\dot{p}_{k-i} < 0$ where it exists.

Now consider $t < t^*_K$. If $(l-1)\Delta \le t < t^*_K < l\Delta$, the H-buyer's indifference condition is:

$$\begin{aligned} v_H - p_{K+1}(t) &= e^{-\lambda\left(t^*_K - t\right)}U^{K+1}_{t^*_K} \\ &\quad + \sum_{k=1}^{K}\lambda^k e^{-\lambda(l\Delta - t)}\sum_{i=1}^{k}\frac{(t^*_K - t)^i}{i!}\frac{\left(l\Delta - t^\Delta_K\right)^{k-i}}{(k-i)!}U^{K+1-k}_{l\Delta}; \end{aligned}$$

or if $(l-1)\Delta \le t < l\Delta \le t^*_K$, the condition becomes:

$$v_H - p_{K+1}(t) = \sum_{k=0}^{K} e^{-\lambda(l\Delta - t)}\frac{\lambda^k(l\Delta - t)^k}{k!}U^{K+1-k}_{l\Delta}.$$

The continuation values $U^k_{l\Delta}$ and $U^{K+1}_{t^*_K}$, defined in the same fashion as before, are the expected utilities of an H-buyer if her next attention time is in the next period or at t^*_K, whichever comes first. The analytical expression for $p_{K+1}(t)$ is then obtained using the continuation values in a recursive way.

It is straightforward to show that $p_{K+1}(t)$ is continuous at t_K^*. In addition, we have

$$\dot{p}_{K+1}(t) = -\lambda (p_K(t) - p_{K+1}(t)) \text{ for } t < t_K^*. \tag{B.6}$$

By construction, it is immediate that $p_{K+1}(t) < p_K(t)$, $\forall t < 1$, hence $\dot{p}_{K+1}(t) < 0$ and $\ddot{p}_{K+1}(t) = -\lambda^2 (p_{K-1} - p_{K+1}) < 0$ where differentiable.

Second, we show some properties of the H-buyers' reservation price. The results are summarized in the following lemma.

Lemma B.1. *For $t < t_k^*$, $\dot{p}_{k+1} - \dot{p}_k < 0$ where $k = \{1, 2, ...K\}$.*

Proof. We solve the closed-form solution of $p_{k+1} - p_k$. Simple algebra implies that

$$\dot{p}_{k+1}(t) - \dot{p}_k(t) = \lambda (p_{k+1} - p_k) + \lambda (p_{k-1} - p_k),$$

which is equivalent to

$$\begin{aligned} & [\dot{p}_{k+1}(t) - \dot{p}_k(t) - \lambda (p_{k+1} - p_k)] e^{-\lambda t} \\ = \; & \frac{d}{dt}\left[(p_{k+1} - p_k) e^{-\lambda t}\right] = -\lambda (p_k - p_{k-1}) e^{-\lambda t}. \end{aligned}$$

Recursively, we have

$$\frac{d^k}{dt^k}\left[(p_{k+1} - p_k) e^{-\lambda t}\right] = -(-\lambda)^k \frac{v_H - v_L}{M+1} e^{-\lambda},$$

so

$$p_{k+1} - p_k = -(-\lambda)^k \frac{v_H - v_L}{M+1} e^{-\lambda} \frac{t^k}{k!} e^{\lambda t} + \sum_{i=1}^{k} C_i \frac{t^{k-i}}{(k-i)!} e^{\lambda t}$$

where C_i is a constant number for each i, and

$$\begin{aligned} \dot{p}_{k+1} - \dot{p}_k \;&=\; -(-\lambda)^k \frac{v_H - v_L}{M+1} e^{-\lambda} \frac{t^{k-1}}{(k-1)!} e^{\lambda t} \\ &\quad + \sum_{i=1}^{k-1} C_i \frac{t^{k-i-1}}{(k-i-1)!} e^{\lambda t} + (\lambda + \rho)(p_{k+1} - p_k) \\ &=\; (p_k - p_{k-1}) + \lambda (p_{k+1} - p_k) \\ &\quad + (\lambda + 1)(-\lambda)^{k-1} \frac{v_H - v_L}{M+1} e^{-\lambda} \frac{t^{k-1}}{(k-1)!} e^{\lambda t}. \end{aligned}$$

Hence, when $k \in \{2, 4, 6, 8, ...\}$, we have $\dot{p}_{k+1} - \dot{p}_k < 0$. By the same logic, we have

$$p_2 - p_1 = \frac{\lambda (v_H - v_L)}{M+1} e^{-\lambda(1-t)} t + e^{\lambda t} C_1 < 0$$

$$\frac{d^{k-1}}{dt^{k-1}}\left[\left(p_{k+1}-p_k\right)e^{-\lambda t}\right]=\left(-\lambda\right)^{k-1}\left[\frac{\lambda\left(v_H-v_L\right)}{M+1}e^{-\lambda}t+C_1\right]$$

so

$$p_{k+1}-p_k=\left(-\lambda\right)^{k-1}\left[\frac{\lambda\left(v_H-v_L\right)}{M+1}e^{-\lambda}\frac{t^{k-1}}{(k-1)!}+C_1\frac{t^{k-2}}{(k-2)!}\right]e^{\lambda t}+\sum_{i=2}^{k}C_i\frac{t^{k-i}}{(k-i)!}e^{\lambda t}$$

and

$$\begin{aligned}\dot{p}_{k+1}-\dot{p}_k &= \lambda\left(p_{k+1}-p_k\right)+\left(p_k-p_{k-1}\right)\\ &\quad -\left(\lambda+1\right)\left(-\lambda\right)^{k-2}\frac{t^{k-3}}{(k-2)!}\left[\frac{\lambda\left(v_H-v_L\right)}{M+1}e^{-\lambda}t+C_1\left(k-2\right)\right]e^{\lambda t}\end{aligned}$$

hence when $k\in\{3,5,7,9...\}$, $\dot{p}_{k+1}-\dot{p}_k<0$. In short, $\dot{p}_{k+1}-\dot{p}_k<0$ for any $k\in\mathbb{N}$. □

Next, we consider the problem faced by the seller in which he chooses the fire sale time t_K^*, but is forced to post $p_{K+1}(t)$ when $t<t_K$ and $K(t)=K+1$, and the seller's problem is to choose the optimal deal time.

$$\Pi_{K+1}(t)=\max_{t_K}\int_t^{t_K}e^{-\lambda(s-t)}\lambda\left[p_{K+1}(s)+\Pi_K(s)\right]ds+e^{-\lambda(t_K-t)}\Pi_c(t_K),\tag{B.7}$$

where the continuation payoff $\Pi_c(t_K)$ is given as follows

$$\Pi_c(t_K)=\begin{cases}v_L+\Pi_K(t_K) & t_K<t_{K-1}^*\\ iv_L+\Pi_{K+1-i}(t_K) & t\in[t_{K+1-i}^*,t_{K-i}^*)\end{cases}$$

for $i=2,3,...K-1$. In equilibrium, the H-buyers' belief is correct, so the seller's optimal choice is indeed $t_k^*(\Delta)$. The first order derivative to t_K is

$$\begin{aligned}&e^{-\lambda(t_K-t)}\lambda[p_{K+1}(t_K)-v_L]+e^{-\lambda(t_K-t)}\dot{\Pi}_K(t_K)\\ =\;&\lambda e^{-\lambda(t_K-t)}\left[p_{K+1}(t_K)-v_L-p_K(t_K)+\Pi_K(t_K)-\Pi_{K-1}(t_K)\right]\end{aligned}$$

At t_k^*, we have

$$p_{K+1}(t_k^*)-v_L=p_K(t_k^*)+\Pi_{K-1}(t_k^*)-\Pi_K(t_k^*).$$

Third, we show that there is a unique t_K^* that determines the auxiliary equilibrium. At t_K^*, we have $p_{K+1}(t_K^*)-v_L-p_K(t_K^*)+\Pi_K(t_K^*)-\Pi_{K-1}(t_K^*)=v_L+\Pi_K(t_K^*)-\Pi_{K-1}(t_K^*)-\frac{p_K(t_K^*)-v_L}{M+1}$. Let

$$f_K^1(t)=v_L+\Pi_K(t)-\Pi_{K-1}(t)-\frac{p_K(t)-v_L}{M+1}\text{ for }t\in[t_k^*,t_{K-1}^*)$$

Similar to the two-unit case, a simple observation is that $\lim_{t\to t^*_{K-1}} \Pi_K(t) - \Pi_{K-1}(t) \to v_L$ and $\lim_{t\to t^*_{K-1}} \left[\mathbb{E}_{\tilde{t}|t}\left[e^{-\lambda(\tilde{t}-t)}p_K(t)\right] - v_L\right] > 0$ and both $\Pi_K(t) - \Pi_{K-1}(t)$ and $p_K(t)$ are continuous function, so $f_K(t) < 0$ for t close to t^*_{K-1}. If $f_K(t) < 0$ for any $t \in [0, t^*_{K-1})$, we claim that $t^*_k = 0$. Otherwise, we let $t^*_k = \sup\{t|t \leq t_{K-1},\, f_K(t) = 0 \text{ and } \exists\varepsilon > 0 \text{ s.t. } f_K(t') > 0$ for $t' \in (t-\varepsilon, t)\}$. By construction, for $t \in \left(t^*_k, t^*_{K-1}\right)$, the optimal solution of (B.7) is t, and when $t < t^*_k$, the seller prefers t^*_k to any $t_K \in \left(t^*_k, t^*_{K-1}\right)$. What about $t_K \in \left(t^*_{K-i}, t^*_{K-i-1}\right)$ for $i = 1, 2, ..K-2$ and $t_K \in (t^*_1, 1)$? The first derivative of the seller's objective function is given by

$$p_{K+1}(t_K) + \Pi_K(t_K) - iv_L - \Pi_{K-i}(t_K) - \left[p_{K-i}(t) + \Pi_{K-i-1} - \Pi_{K-i}\right]$$

for $t_K \in \left(t^*_{K-i}, t^*_{K-i-1}\right)$. By construction of p_k, we have

$$p_{K+1}(t_K) = \frac{i+1}{M+1}v_L + \frac{M-i}{M+1}p_{K-i}(t),$$

where $\tilde{t}$ is the H-buyer's next regular attention time. Let

$$\begin{aligned}
f^i_K(t) &= \Pi_K(t_K) - iv_L - \Pi_{K-i-1}(t_K) + \left[p_{K+1}(t_K) - p_{K-i}\right] \\
&= \left[\Pi_K(t_K) - iv_L - \Pi_{K-i-1}(t_K)\right] \\
&\quad + \left\{\frac{i+1}{M+1}v_L + \frac{M-i}{M+1}p_{K-i}(t) - p_{K-i}(t)\right\} \\
&= \left[\Pi_K(t_K) - iv_L - \Pi_{K-i-1}(t_K)\right] + \frac{i+1}{M+1}\left[v_L - p_{K-i}(t)\right].
\end{aligned}$$

And by construction of Π_k, $\Pi_K(t_K) - iv_L - \Pi_{K-i-1}(t_K) = 0$ for $t_K \in (t_{K-i}, t_{K-i-1})$. So $f^i_K < 0$. As a result, the seller, holding $K+1$ units, prefers to sell the first unit via a fire sale at t^*_k to any $t_K \in (t^*_k, 1]$.

Now let us verify whether, at $t < t^*_k$, the seller's optimal choice is t^*_k if $K(t) = K+1$. The first derivative is given by

$$p_{K+1}(t) - v_L - p_K(t) + \Pi_K(t) - \Pi_{K-1}(t)$$

and we know that it equals zero at t^*_k. We want to show that, for any $t < t^*_k$, the first derivative is positive. The reason is simply that both $\dot{p}_{K+1}(t) - \dot{p}_K(t)$ and $\dot{\Pi}_K(t) - \dot{\Pi}_{K-1}(t)$ are negative. The first term is proved in Lemma B.1, and the second term is shown as follows. We know that $\dot{\Pi}_2(t) - \dot{\Pi}_1(t) < 0$. Now at t^*_k, $\dot{\Pi}_K(t^*_k) - \dot{\Pi}_{K-1}(t^*_k) = 0$, and the $\lim_{t\searrow t^*_k}\left[\ddot{\Pi}_K(t) - \ddot{\Pi}_{K-1}(t)\right] = -\lambda\left[\dot{p}_K(t) - \dot{p}_{K-1}(t) + \dot{\Pi}_{K-1}(t) - \dot{\Pi}_{K-2}(t)\right] >$

0. Hence $\dot{\Pi}_K(t) - \dot{\Pi}_{K-1}(t) < 0$ for $t \in (t_k^* - \varepsilon, t_k^*)$ where ε is small but positive. If $\dot{\Pi}_K(t) - \dot{\Pi}_{K-1}(t) > 0$ for some t, by continuity of $\dot{\Pi}_K(\cdot) - \dot{\Pi}_{K-1}(\cdot)$, there must be a $\hat{t}$ s.t. $\dot{\Pi}_K(\hat{t}) - \dot{\Pi}_{K-1}(\hat{t}) = 0$ and $\dot{\Pi}_K(t) - \dot{\Pi}_{K-1}(t) < 0$ for any $t \in (\hat{t}, t_k^*)$. However, $\ddot{\Pi}_K(\hat{t}) - \ddot{\Pi}_{K-1}(\hat{t}) > 0$, which is a contradiction!

In short, the equilibrium of the auxiliary problem uniquely exists. Similar to the two-unit case, off the path of play, if the fire sale is postponed such that (1) $K(t) = k$ and (2) t and t_{k-2}^* are in the same period, the H-buyer's reservation price is lower than $p_k(t)$ and $p_{k-1}(t)$ when $K(t) = k$ and $k-1$ respectively. However, the seller's profit by running the fire sale in such a period is strictly less than that in the auxiliary problem. Hence, it is strictly dominated. Consequently, in the real problem, the seller does not have any incentive to choose a deal time later than t_{k-1}^* when $K(t) = k$.

B.4.2 Verification of the Conjecture

First, we verify that in the candidate equilibrium, two sales are not in the same period.

Lemma B.2. *When Δ is small, and $t_k^*, t_{k+1}^* > \Delta$, $t_k^* - t_{k+1}^* > \Delta$, for all $k \in \mathbb{N}$*

Proof. If $t_k^*, t_{k+1}^* > \Delta$, they are both interior solutions. By construction, we have that $t_k^* > t_{k+1}^*$. We claim that $t_k^*, t_{k+1}^*$are not in the same period when Δ is small. Suppose not, we have $t_k^* - t_{k+1}^* \leq \Delta$. And at t_k^*, we have

$$\begin{aligned}
\Pi_{k+1}(t_k^*) &= \Pi_k(t_k^*) + v_L, \dot{\Pi}_{k+1}(t_k^*) = \dot{\Pi}_k(t_k^*), \\
\dot{\Pi}_{k+1}(t_k^*) &= -\lambda\left[p_{k+1}(t_k^*) + \Pi_k(t_k^*) - \Pi_{k+1}(t_k^*)\right], \\
\dot{\Pi}_k(t_k^*) &= -\lambda\left[p_k(t_k^*) + \Pi_{k-1}(t_k^*) - \Pi_k(t_k^*)\right],
\end{aligned}$$

so

$$p_{k+1}(t_k^*) - v_L = p_k(t_k^*) + \Pi_{k-1}(t_k^*) - \Pi_k(t_k^*).$$

but at t_{k+1}^*, we have

$$\begin{aligned}
p_{k+2}(t_{k+1}^*) - v_L &= p_{k+1}(t_{k+1}^*) + \Pi_k(t_{k+1}^*) - \Pi_{k+1}(t_{k+1}^*) \\
&= p_{k+1}(t_{k+1}^*) - v_L + \underbrace{\int_{t_{k+1}^*}^{t_k^*} \left[\dot{\Pi}_k(s) - \dot{\Pi}_{k+1}(s)\right] ds}_{O(\Delta)}.
\end{aligned}$$

However, $p_{k+1}(t_k^*) - p_{k+2}(t_{k+1}^*)$ is bounded away from zero for any Δ. So, when $\Delta \to 0$, we have a contradiction! Thus, when Δ is small, we have the desired result. $\square$

Second, we verify that the equilibrium path $\{p_k(t), t^*_{k-1}\}_{k=1}^{K+1}$ can be supported as a equilibrium. The proof is almost identical to the two-unit case. In any histories with $K(t) = k$, the seller posts either $p_k(t)$ or v_L, thus $\Phi_0(t) = 1$. Since t^*_k is the optimal fire sale time when $K(t) = k$ and $\Phi_0(t) = 1$, the seller has no incentive to deviate. So the conjecture equilibrium is indeed an equilibrium.

Uniqueness. Similar to the $K = 2$ case. In any period, there is no non-selling time intervals. Given that, the only possible equilibrium is the equilibrium we proposed. *Q.E.D.*

References

[1] Ambrus, A. and Lu, S. (2010): "A Countinuous-Time Model of Multilateral Bargaining," *mimeo*, Duke University.

[2] Armstrong, M. and Zhou, J. (2011): "Exploding Offers and Buy-Now Discounts," *mimeo*, New York University.

[3] Ausubel, L. and Deneckere, R. (1989): "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, Vol.57, pp. 511-531.

[4] Aviv, Y. and Pazgal, A. (2008): "Optimal Pricing of Seasonal Products in the Presence of Forward-Looking Consumers," *Manufacturing & Service Operations Management*, Vol. 10, pp. 339-359.

[5] Bergin, J. and MacLeod, B. (1993): "Continuous Time Repeated Games," *International Economic Review*, Vol. 34, pp. 21-37.

[6] Besanko, D. and Winston, W. (1990): "Optimal Price Skimming by a Monopolist Facing Rational Consumers," *Management Science*, Vol.36, pp. 555-567.

[7] Board, S. (2008): "Durable-Goods Monopoly with Varying Demand," *Review of Economic Studies*, Vol. 75, pp. 391-413.

[8] Board, S. and Skrzypacz, A. (2010): "Revenue Management with Forward Looking Buyers," *mimeo*, UCLA.

[9] Biehl, A. (2001): "Durable-Goods Monopoly with Stochastic Values," *RAND Journal of Economics*, Vol. 32, pp. 565-577.

[10] Bulow, J. (1985): "Durable-goods monopolists," *Journal of Political Economy*. Vol.90, pp. 314-332.

[11] Conlisk, J., Gerstner, E. and Sobel, J. (1984): "Cyclic Pricing by a Durable Goods Monopolist," *Quarterly Journal of Economics*, Vol. 99, pp. 489–505

[12] Courty, P. and Li, H. (2000): "Sequential Screening," *Review of Economic Studies*, Vol. 67, pp. 697-717.

[13] Chen, C. (2012): "Name Your Own Price at Priceline.com: Strategic Bidding and Lockout Periods," *Review of Economic Studies*, forthcoming.

[14] Davis, P. (1994): "Airline Ties Profitability Yield to Management," *SIAM News*, Vol. 27(5).

[15] Deb, R. (2010), "Intertemporal Price Discrimination with Stochastic Values?" *mimeo* University of Toronto.

[16] Deb, R. and Said, M. (2012): "Dynamic Screening With Limited Commitment," *mimeo* University of Toronto.

[17] Deneckere, R. and Peck, J. (2012): "Dynamic Competition with Random Demand and Costless Search: A Theory of Price Posting," *Econometrica*, Vol. 80, pp. 1185-1247.

[18] Dudine, P., Hendel, I. and Lizzeri, A. (2006): "Storable Good Monopoly: The Role of Commitment," *American Economic Review*, Vol. 96, pp. 1706-1719.

[19] Ely, J., Garrett, D., and Hinnosaar, T. (2012): "Overbooking," *mimeo*, Northwestern University.

[20] Friend, S., and Walker, P. (2001): "Welcome to the New World of Merchandising," *Harvard Business Review*, Vol. 79, 133-141.

[21] Fu, X., Gautier, P., and Watanabe, M. (2012): "Overbooking in Markets with Rationing," *mimeo*, VU Amsterdam.

[22] Fuchs, W. and Skrzypacz, A. (2010): "Bargaining with Arrival of New Traders," *American Economic Review*, Vol. 100, pp. 802-836.

[23] Fuchs, W. and Skrzypacz, A. (2011): "Bargaining with Deadlines and Private Information," *mimeo*, UC Berkeley.

[24] Fudenberg, D., Levine D. and Tirole, J. (1985): "Infinite-horizon Models of Bargaining with One-Sided Incomplete Information," In *Game Theoretic Models of Bargaining*, ed. Roth, A., 73-98. Cambridge, MA: Cambridge University Press.

[25] Fudenberg, D. and Tirole, J. (1983): "Sequential Bargaining with Incomplete Information," *Review of Economic Studies*, Vol. 50, pp. 221-247.

[26] Gul, F., Sonnenschein, H. and Wilson, R. (1986): "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, Vol. 39, pp. 155-190.

[27] Garrett, D. (2011): "Durable Goods Sales with Changing Values and Unobservable Arrival Dates," *mimeo*, Northwestern University.

[28] Gershkov, A. and Moldovanu, B. (2009): "Dynamic Revenue Maximization with Heterogeneous Objects: A Mechanism Design Approach," *American Economic Journal: Microeconomics*, Vol. 1, pp. 168–198.

[29] Hörner, J. and Samuelson, L. (2011): "Managing Strategic Buyers," *Journal of Political Economy*, Vol. 119, pp. 379-425.

[30] Hörner, J. and Vieille, N. (2009): "Public vs. Private Offers in the Market for Lemons," *Econometrica*, Vol.77, pp. 29-69.

[31] Huang, C. and Li, F. (2012): "Bargaining While Learning about New Arrivals," *mimeo*, University of Pennsylvania.

[32] Jerath, K., Netessine, S. and Veeraraghavan, S. (2010): "Revenue Management with Strategic Customers: Last-Minute Selling and Opaque Selling," *Management Science*, Vol. 56, pp. 430-448.

[33] Kamada, Y. and Kandori, M., "Revision Games," *mimeo*, Harvard University.

[34] Kaya, A., and Liu, Q. (2012): "Transparency and Price Formation in Bargaining," *mimeo*, University of Iowa.

[35] Kim, L. (2012): "Information about Sellers' Past Behavior in the Market for Lemons," *mimeo*, University of Iowa.

[36] Kreps, D. and Scheinkman, J. (1983): "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics*, Vol. 14, pp. 326-337.

[37] Lazarev, J. (2012): "The Welfare Effects of intertemporal Price Discrimination: An Empirical Analysis of Airline Pricing in U.S. Monopoly Markets," *mimeo*, New York University.

[38] Lazear, E. (1986): "Retail Pricing and Clearance Sales?" *American Economic Review*, Vol. 76, 14-32.

[39] Li, A. (2012): "Selling Storable Goods to a Dynamic Population of Buyers: A Mechanism Design Approach," *mimeo*, Stanford University.

[40] Maskin, E. and Tirole, J. (1988): "A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles," *Econometrica*, Vol. 56, pp. 571-599.

[41] Mason, R. and Valimaki, J. (2011): "Learning about the Arrival of Sales," *Journal of Economic Theory*, forthcoming.

[42] McAfee, P. and te Velde, V. (2007): "Dynamic Pricing in the Airline Industry," *Handbook on Economics and Information Systems*, ed. Hendershott, T. 527–70. Amsterdam, The Netherlands: Elsevier.

[43] McAfee, P. and Vincent, D. (1997): "Sequential Optimal Auctions," *Games and Economic Behavior*, Vol. 18, pp. 246-276.

[44] McAfee, P. and Wiseman, T. (2008): "Capacity Choice Counters the Coase Conjecture," *Review of Economic Studies*, Vol.75, pp. 317-332.

[45] Mierendorff, K. (2011a): "The Dynamic Vickrey Auction," *mimeo*, University of Zurich.

[46] Mierendorff, K. (2011b): "Optimal Dynamic Mechanism Design with Deadlines," *mimeo*, University of Zurich.

[47] Milgrom, P. (1981): "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, Vol. 12, pp. 380-391.

[48] Möller, M., and Watanabe, M. (2009): "Advance Purchase Discounts versus Clearance Sales," *Economic Journal*, Vol. 120, pp. 1125-1148.

[49] Nocke, V. and Peitz, M. (2007): "A Theory of Clearance Sales," *Economic Journal*, Vol. 117, pp. 964-990.

[50] Pai, M. and Vohra, R. (2010): "Optimal Dynamic Auctions," *mimeo*, University of Pennsylvania.

[51] Perry and Reny (1993): "A Non-Cooperative Bargaining Model with Strategically Timed Offers", *Journal of Economic Theory*, Vol. 59, pp. 50-77.

[52] Said, M. (2012): "Auctions with Dynamic Populations: Efficiency and Revenue Maximization," *Journal of Economic Theory*, forthcoming.

[53] Shneyerov, A. (2012): "Revenue Management by a Patient Seller," *mimeo*, Concordia University.

[54] Simon, L. and Stinchcombe, M. (1989): "Extensive Form Games in Continuous Time: Pure Strategies," *Econometrica*, Vol. 57, pp. 1171-1214.

[55] Skreta, V. (2006): "Sequentially Optimal Mechanisms," *Review of Economic Studies*, Vol. 73, pp. 1085-1111.

[56] Skreta, V. (2011): "Optimal Auction Design under Non-Commitment," *mimeo*, New York University.

[57] Sobel, J. (1984): "The Timing of Sales," *Review of Economic Studies*, Vol. pp. 353-368.

[58] Sobel, J. (1991): "Durable Goods Monopoly with Entry of New Consumers," *Econometrica*, Vol. 59, pp. 1455-1485.

[59] Stokey, N. (1979): "Intertemporal Price Discrimination," *Quarterly Journal of Economics*, Vol. 93, pp. 355-371.

[60] Stokey, N. (1981): "Rational Expectations and Durable Godds Pricing," *The Bell Journal of Economics*, Vol. 12, pp. 112-128.

[61] Su, X. (2007): "Intertemporal Pricing with Strategic Customer Behavior," *Management Science*, Vol. 53, pp. 726-741.

[62] Sweeting, A. (2012): "Price Dynamics in Perishable Goods Markets: The Case of Secondary Markets for Major League Baseball Tickets," *Journal of Political Economy*, forthcoming.

[63] Talluri, K. and Ryzin, G. (2004): *The Theory and Practice of Revenue Management*. Springer.

[64] Varian, H. (1980): "A Model of Sales," *American Economic Review*, Vol. 70, pp. 651-659.

[65] Veldkamp, L. (2011): *Information Choice in Macroeconomics and Finance*. Princeton University Press.

[66] Wang, R. (1993): "Auctions versus Posted-Price Selling," *American Economic Review*, Vol. 83, pp. 838-851.